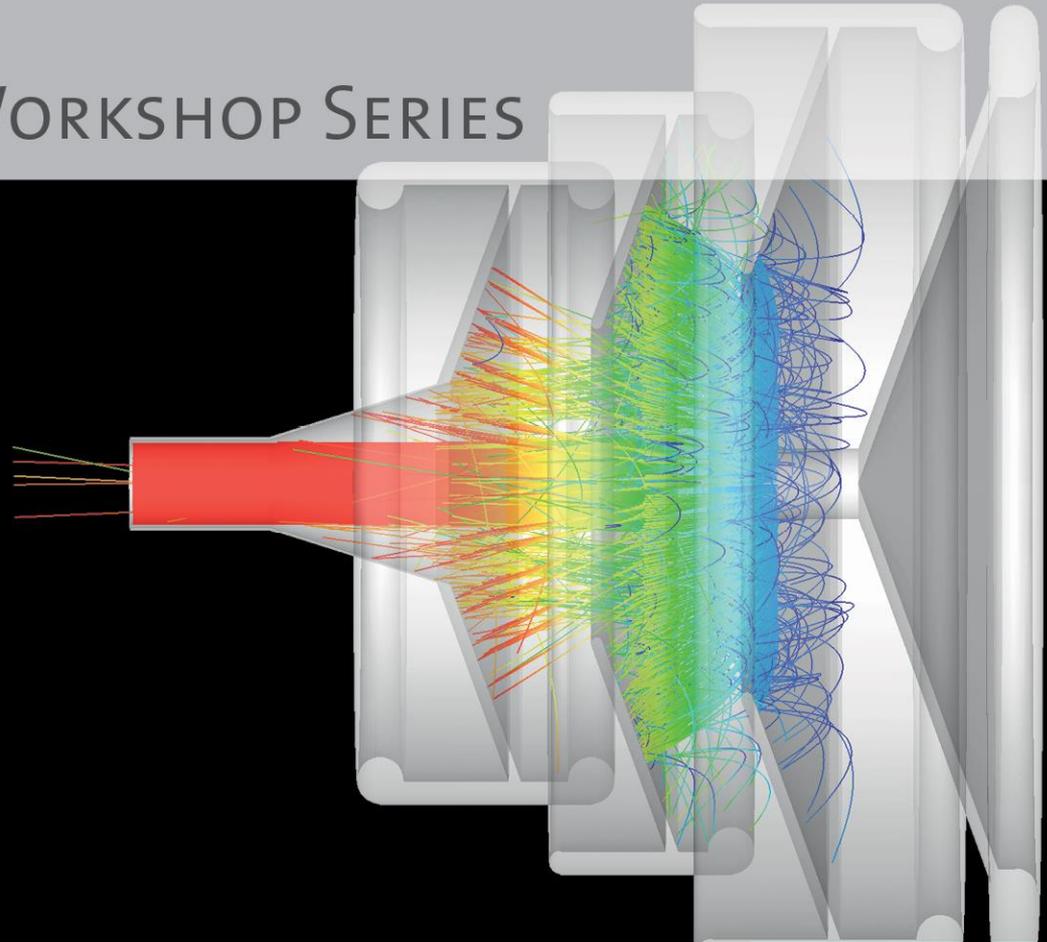


CST STUDIO SUITE®

PARTICLE DYNAMICS WORKSHOP SERIES

Simulations with EM Studio and Particle Studio



Agenda

- **EM/PS Solver Overview**
- **Magnets**
- **Cavity Simulations**
 - Eigenmodes & PostProcessing
- **Wakefield Analysis**
- **Full Particle Dynamics**

CST STUDIO SUITE®

Solver Overview



RF Fields		
	Transient	<ul style="list-style-type: none"> ▪ broadband ▪ arbitrary time signals
	Eigenmode	<ul style="list-style-type: none"> ▪ strongly resonant structures ▪ narrow band cavities
Static Fields		
	E- Static	<ul style="list-style-type: none"> ▪ static electric problems
	M- Static	<ul style="list-style-type: none"> ▪ static magnetic problems
Particle Solvers		
	Tracking	<ul style="list-style-type: none"> ▪ Solver for static simulations such as electron guns or mass spectrometer
	Particle In Cell	<ul style="list-style-type: none"> ▪ Selfconsistent transient field and particle solver including full space charge effects
	Wakefield	<ul style="list-style-type: none"> ▪ Transient solver with special beam excitation (predefined fixed straight beam path)



Electrostatics

Sources:

- Potentials on conductors (fixed or floating)
- Boundary potentials (fixed or floating)
- Homogeneous volume and surface charges
- Charges on PEC solids

Postprocessing:

- Capacitance matrix
- Force calculation

Additional features:

- Adaptive meshing
- Optimizer
- Open boundaries



Magnetostatics

Sources:

- Current paths
- Current coils
- Permanent magnets
- Homogeneous magnetic source fields
- Stationary current field

Postprocessing:

- Inductance matrix
- Torque and force calculation

Additional features :

- Adaptive meshing
- Optimizer
- Nonlinear materials
- Open boundaries

CST EMS Sources



Electric Potential Electric Charge on PEC Electric Charge Distribution Lumped Element

Sources and Loads



Permanent Magnet Coil Current Path Magnetic Source Field Electric Potential Current Port

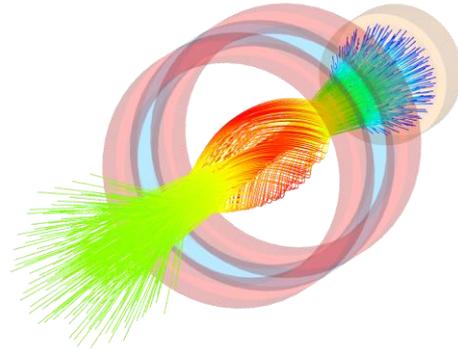
Sources and Loads



Tracking Solver

Algorithm:

1. Calculate electro- and magnetostatic fields
2. Calculate force on charged particles
3. Move particles according to the previously calculated force  Trajectory



$$\frac{d}{dt}(m\vec{v}) = q(\vec{E} + \vec{v} \times \vec{B})$$

Velocity update

$$m^{n+1}\vec{v}^{n+1} = m^n\vec{v}^n + q\Delta t(\vec{E}^{n+1/2} + \vec{v}^{n+1} \times \vec{B}^{n+1/2})$$

$$\frac{d\vec{r}}{dt} = \vec{v}$$

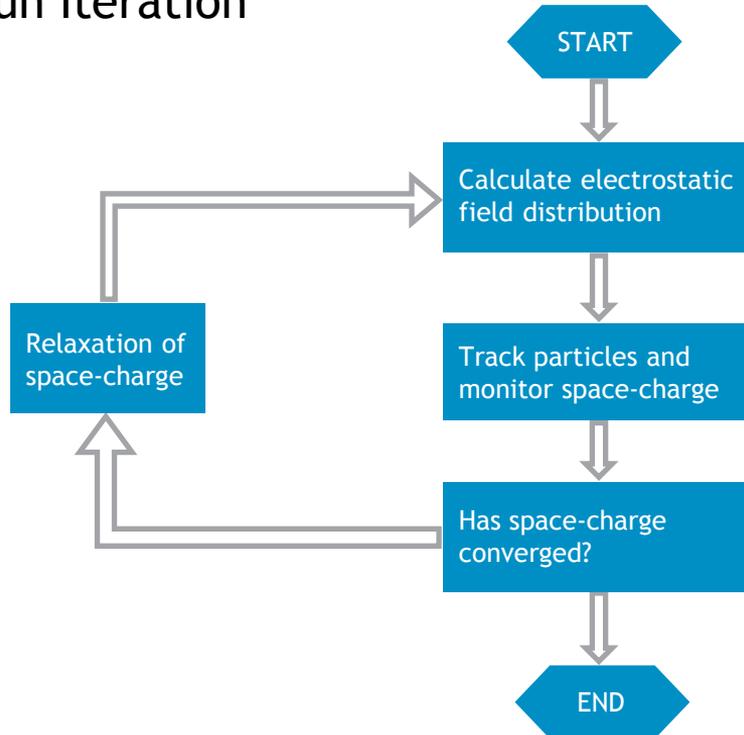
Position update

$$\vec{r}^{n+3/2} = \vec{r}^{n+1/2} + \Delta t\vec{v}^{n+1}$$

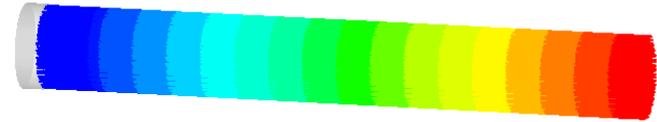
Leap Frog Scheme

Space Charge Effect

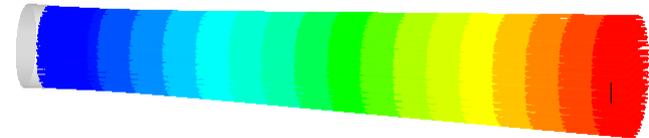
Gun Iteration



Without Gun Iteration:



With Gun Iteration:



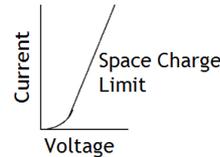
TRK Emission Models

1. Fixed Emission

Applicable for known beams
Input: Current, Beam Voltage

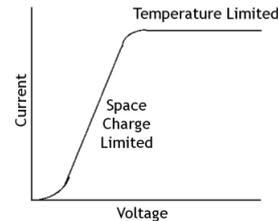
2. Space Charge Limited Emission

Necessary for Electron Guns
Current(Voltage) according to Childs Law



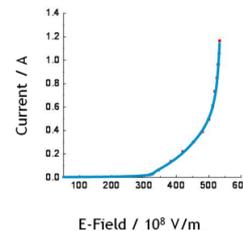
3. Thermionic Emission

Enhances Space Charge Limit
Richardson Dushman Equation



4. Field Induced Emission

Fowler-Nordheim Equation



Particle Interfaces

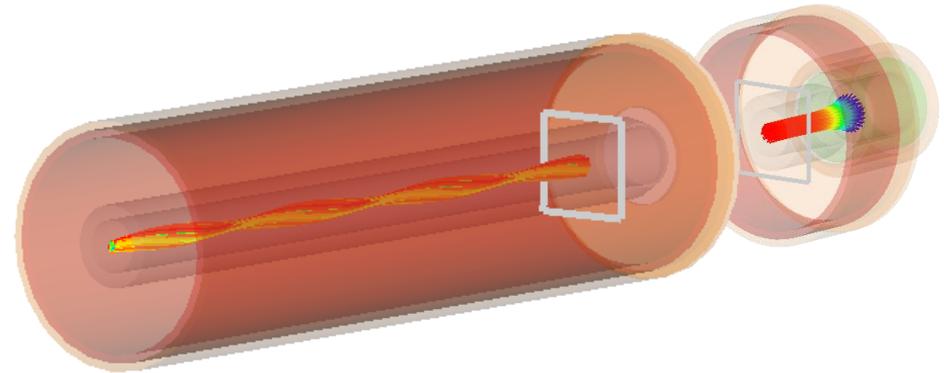
Particle Emission

- According to exported data of previous simulation
- According to ASCII files of known particle distributions

Advantage

- Splitting of large tracking simulations
- Speed up for collector simulation

Gun simulation with
export interface

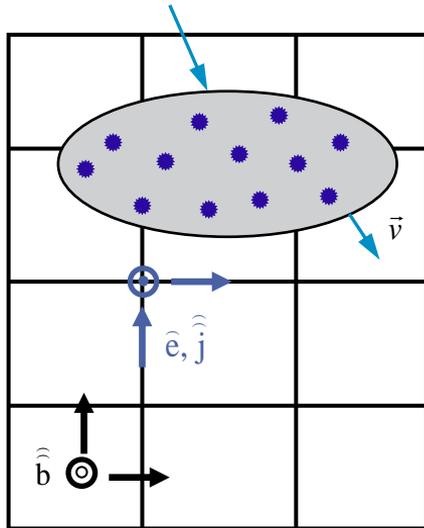


Tracking/PIC simulation
with import interface

Particle in Cell

Self consistent modeling
of collisionfree plasma

Macro charge (e.g. $q=10^6 e^-$)



Relativistic equation of motion

$$\frac{\partial \vec{u}}{\partial t} = \frac{q}{m_0 c} (\vec{E} + \vec{v} \times \vec{B})$$

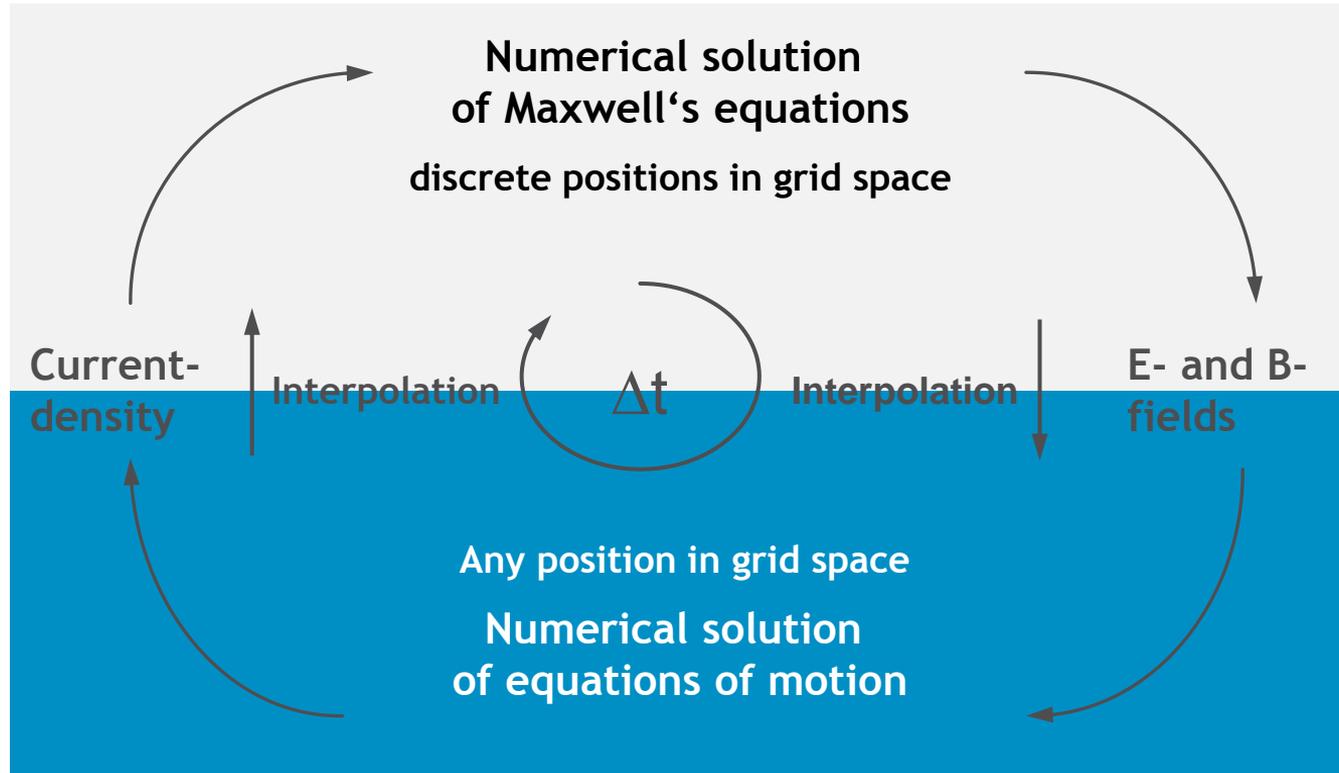
$$\frac{\partial \vec{r}}{\partial t} = \vec{v} \quad \vec{u} = \frac{\vec{p}}{m_0 c} = \frac{m(v) \vec{v}}{m_0 c} = \gamma \cdot \vec{\beta}$$

Current caused by particle motion acts as
source in Maxwell's equations.

$$\text{rot} \vec{H} = \frac{\partial \vec{D}}{\partial t} + \boxed{\vec{J}}; \quad \text{div} \vec{J} = -\frac{\partial \rho}{\partial t}$$

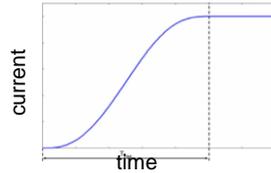
A priori charge conserving algorithm.

Particle in Cell (PIC) Algorithm

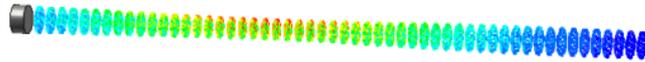


PIC Emission Models

1. DC Emission



2. Emission of a series of Gaussian bunches

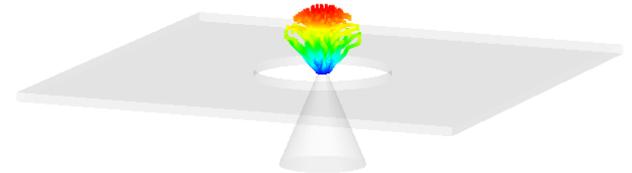
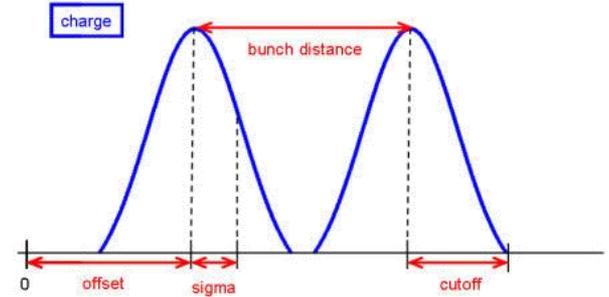


3. Field Emission according to Fowler Nordheim Equation

4. TRK/PIC interfaces for emitting the beam of a specific gun or for loading ASCII files of known particle distributions

5. Secondary electron emission

6. Explosive Emission

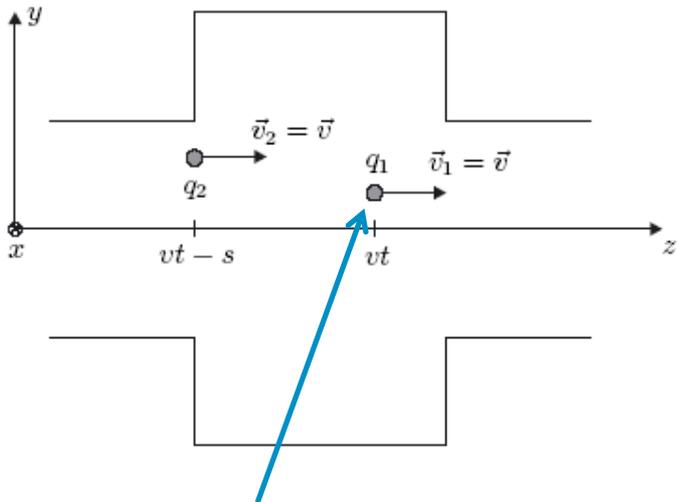




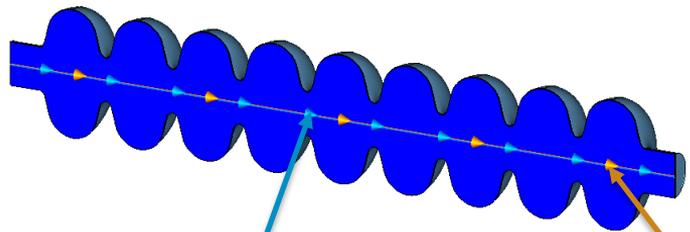
Wakefield Solver

The Wakefield solver computes the wake potential:

$$\vec{W}(x, y, s) = \frac{1}{q_1} \int_{-\infty}^{\infty} \left(\vec{E}(x, y, z, t = \frac{s+z}{v}) + \vec{v} \times \vec{B}(x, y, z, t = \frac{s+z}{v}) \right) dz$$



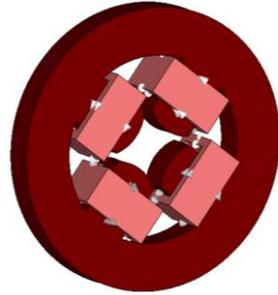
Excitation (pencil beam with longitudinal Gaussian shape)



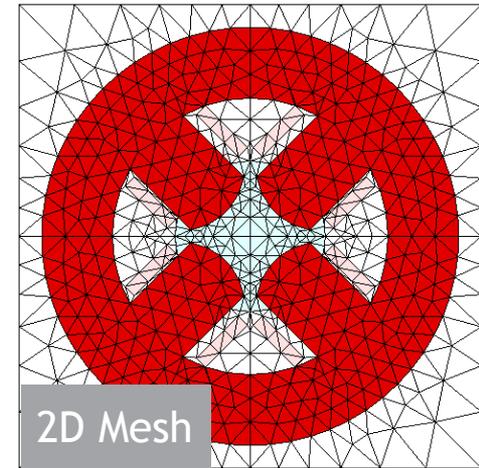
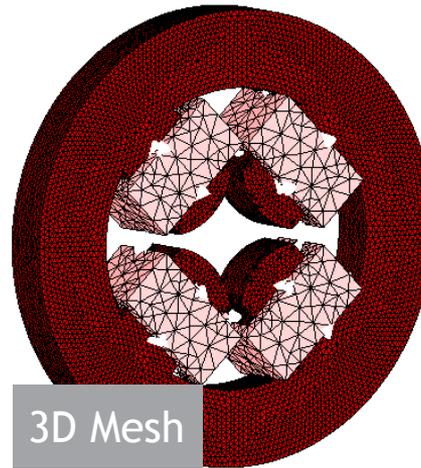
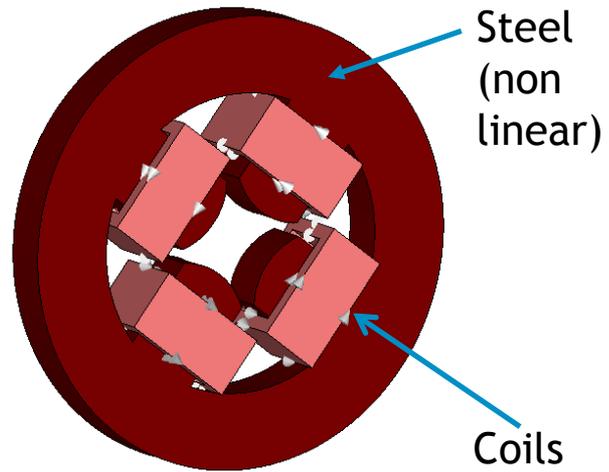
Beam path

Integration path

Magnets



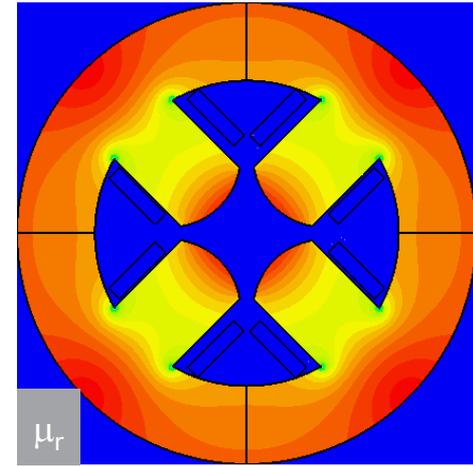
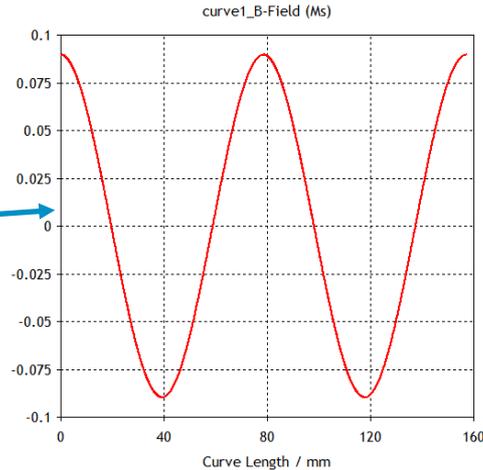
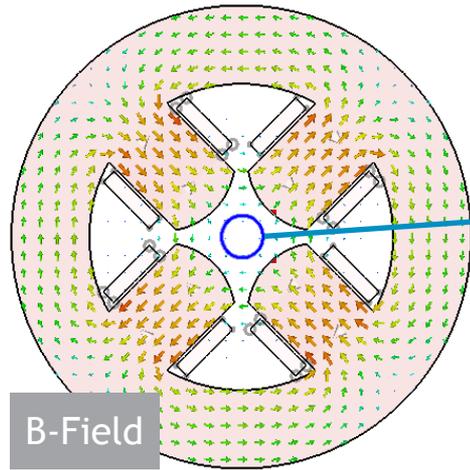
Magnet Design



Quadrupole Magnet:

- Source: Coils/Permanent Magnets
- 3D M-static run
- 2D M-static rotational/translational symmetry possible

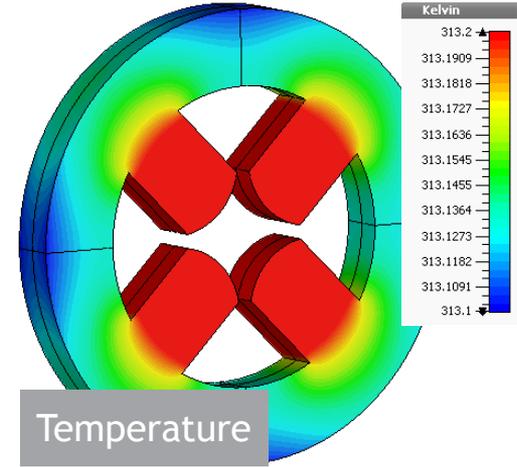
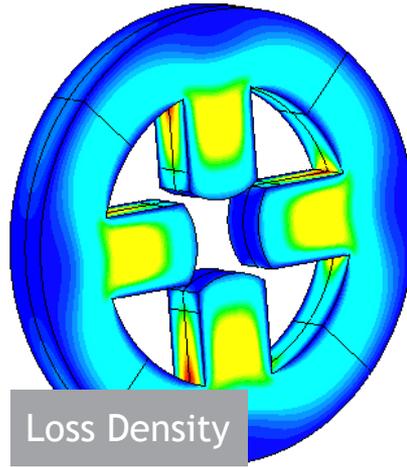
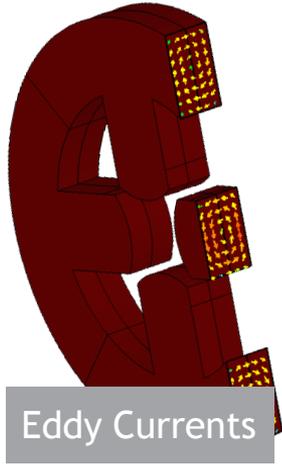
Magnet Design



Results from M-Static:

- Quadrupole like B-Field
- Sinusoidal magnetic field along azimuth
- Pole pieces quite saturated

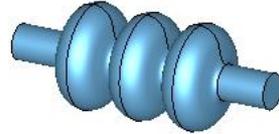
LF-Thermal Coupling



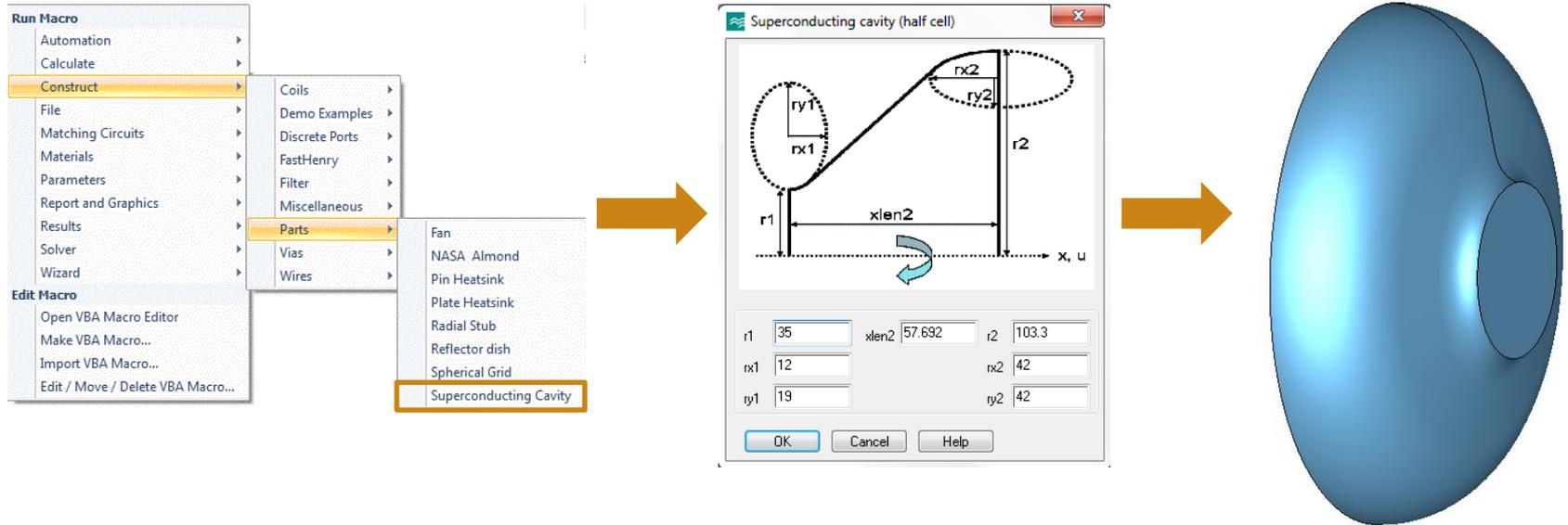
Coupled Simulation:

- Import of LF losses into thermal solver
- Thermal stationary/transient run
- Automization of loop possible via SAM approach

Cavity Simulation

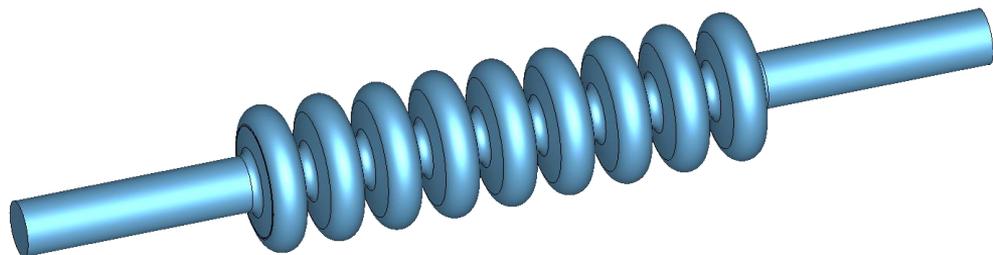


Cavity Construction



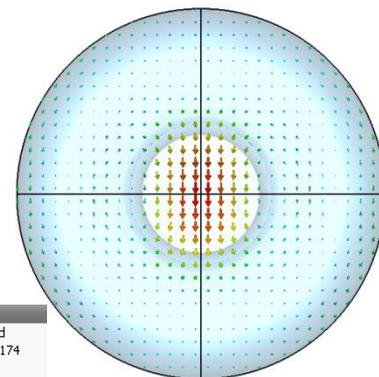
Automatic creation of a cavity part
→ Transformation for multiples

Eigenmode Simulation



Crab cavity courtesy of
Dr. Graeme Burt, Lancaster University

Solver	Advantage	Disadvantage	Mesh Type
AKS	fast	no losses	Hexahedral
JDM	robust	slow	Hexahedral
Default	robust, fast	-	Tetrahedral



Mode 9 (peak)
Type: H-Field
Frequency: 8.129174
Phase: 90

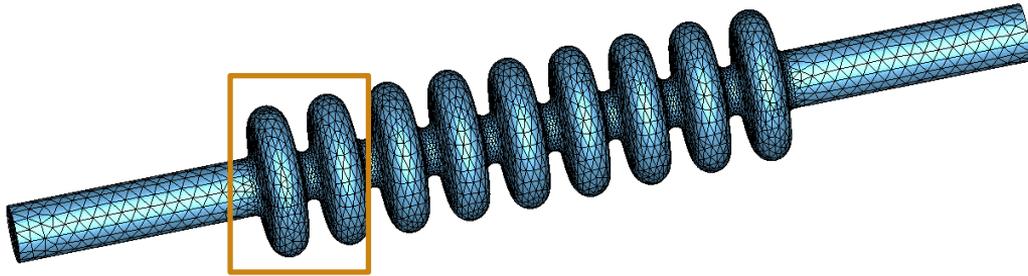
Hexahedral mesh

→ no convergence

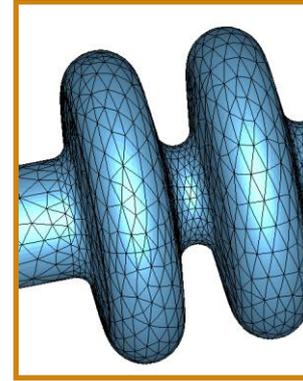
Tetrahedral mesh

→ convergence in < 2 min

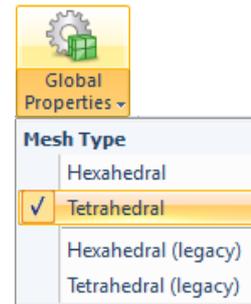
Eigenmode Simulation



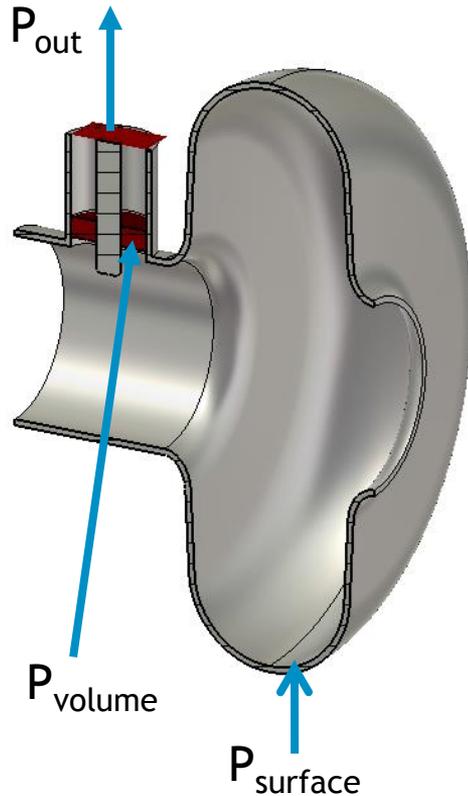
Crab cavity courtesy of
Dr. Graeme Burt, Lancaster University



- Automatic curved elements up to 3rd order applied
- New TET mesh engine for improved mesh distribution



Q-Factor and Loss Calculation



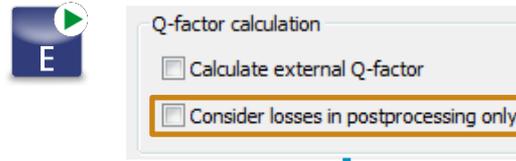
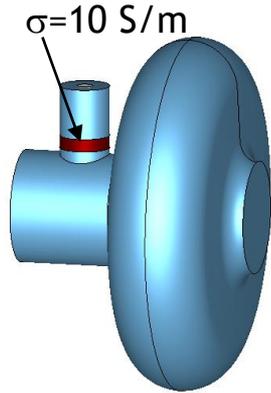
Different Loss Mechanisms:

- Output power through a coupler (P_{out})
- Volume losses in lossy dielectrics (P_{volume})
- Surface losses due to finite metal conductivity ($P_{surface}$)

Superposition for each mode to get Q

$$\begin{aligned}\frac{1}{Q_{loaded_n}} &= \frac{P_{volume} + P_{surface} + P_{out}}{\omega \cdot W} \\ &= \frac{1}{Q_{volume}} + \frac{1}{Q_{surface}} + \frac{1}{Q_{external}}\end{aligned}$$

Volume and Surface Losses



Eigenmode solver results:

Mode	Frequency	Accuracy	
1	0.9136223 GHz	4.515e-007	Q = 1.238e+006

On (Default):
Losses via perturbation method → less resources

Off:
Losses included in solver run → needed if losses change the field pattern

Surface losses are always evaluated with Loss and Q post processing

Normalization to 1J



Q-Factor Calculation

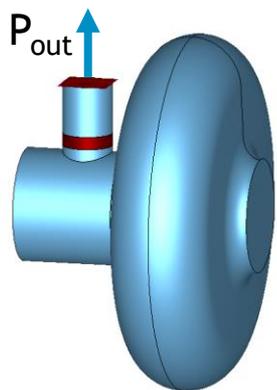
H-Field data: Mode 1

Material/Solid	Conductivity	Mue	Loss/W	Loss/%	Q
***Cond. Enclosure**	5.8000e+007	1	2.3342e+005	98.9	2.4520e+004
PEC	5.8000e+007	1	4.1562e+000	0.00176	1.3771e+009
component1:solid3			3.5993e-002	1.53e-005	1.5901e+011
component1:solid3_1			4.1202e+000	0.00175	1.3891e+009
***Sum of Surface Losses**			2.3342e+005	98.9	2.4519e+004
***Volume Losses**			2.5148e+003	1.07	2.2759e+006
***Sum**			2.3594e+005		2.4258e+004

Buttons: Calculate, Specials..., Export..., Close, Help

Buttons: Modify..., Modify All..., Hide / Unhide, Hide/Unh. All

External Q



Q-factor calculation

Calculate external Q-factor



External Q-factor results:

Mode	Loaded Frequency	External Q
1	0.9136 GHz	1.499e+005

Template Based Postprocessing

General Results

2D and 3D Field Results

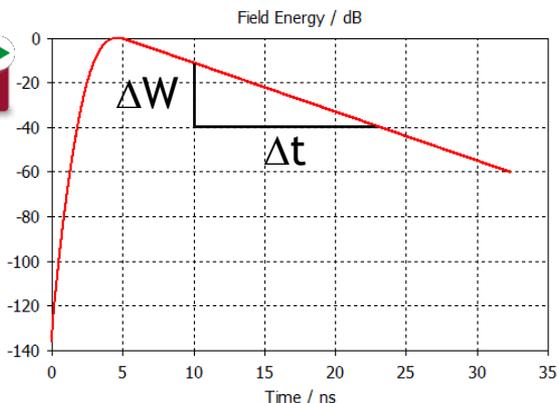
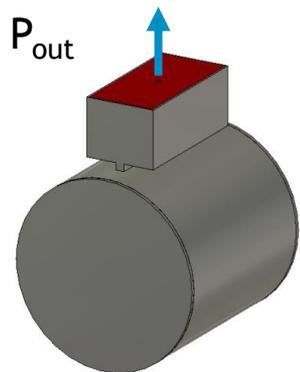
3D Eigenmode Result

3D Eigenmode Result

Result value: Modes: eg 1,3,5-10

Equivalent Input Power (Average) 1 All

(Conductivity taken from Results->Loss and Q-Calculation...)



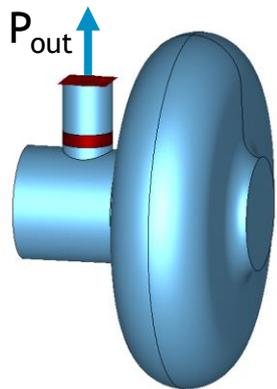
For lossless structures:

$$Q_{\text{external}} = Q_{\text{loaded}}$$

$$\tau = -10 \frac{1}{\frac{\Delta W}{\Delta t} \ln(10)}$$

$$Q_{\text{loaded}} = \tau \cdot \omega_0$$

External Q



Q-factor calculation

Calculate external Q-factor



External Q-factor results:

Mode	Loaded Frequency	External Q
1	0.9136 GHz	1.499e+005

Template Based Postprocessing

General Results

2D and 3D Field Results

3D Eigenmode Result

3D Eigenmode Result

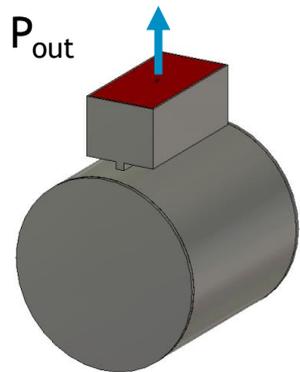
Result value:

Equivalent Input Power (Average)

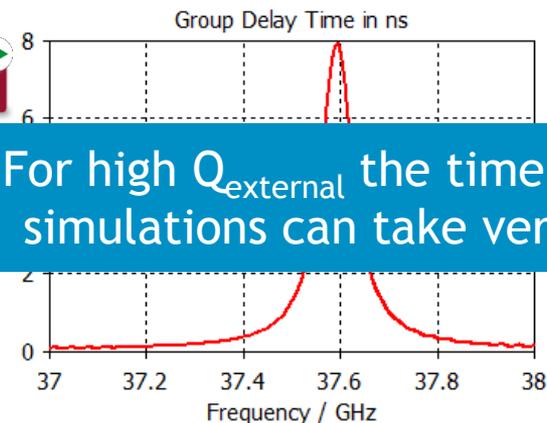
Modes: eg 1,3,5-10

All

(Conductivity taken from Results->Loss and Q-Calculation...)

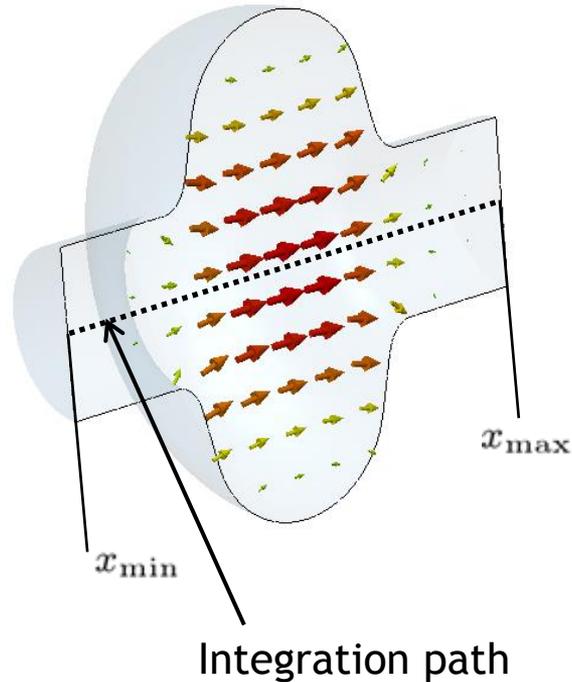


For high Q_{external} the time domain simulations can take very long.



$$Q_{\text{ext.}} = \frac{\omega_0 \Gamma_{D1}}{4}$$

Shunt Impedance, R over Q



Shunt Impedance:

$$R_n := \frac{|U_n|^2}{P_{\text{loss}_n}}$$

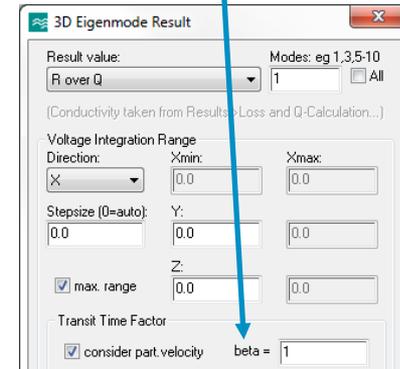
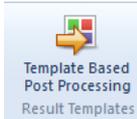
$$U_n := \int_{x_{\min}}^{x_{\max}} E_x(x) \cdot e^{j \frac{\omega_n x}{\beta \cdot c_0}} dx$$

R over Q:

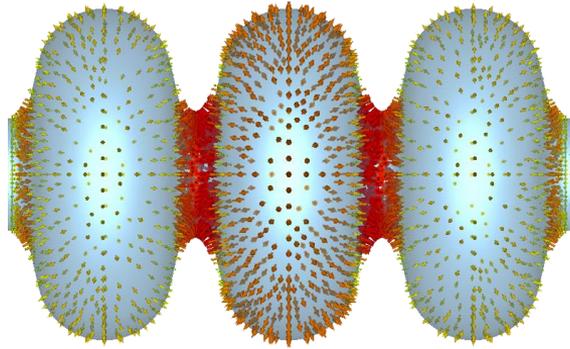
$$\frac{R_n}{Q_n} = \frac{|U_n|^2}{\omega_n \cdot W_n}$$

ω_n = frequency of mode n
 W_n = stored energy of mode n

Integration including
particle velocity in post
processing



Lorentz Force Density



Force Density:

$$P = \frac{1}{4} (\mu_0 H_s^2 - \epsilon_0 E_s^2)$$

Especially interesting
for superconducting
cavities

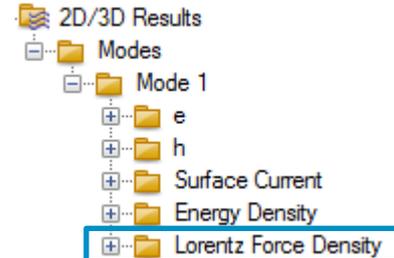


*) Picture from H. Gassot,
Institut de Physique Nucléaire,
Orsay, France

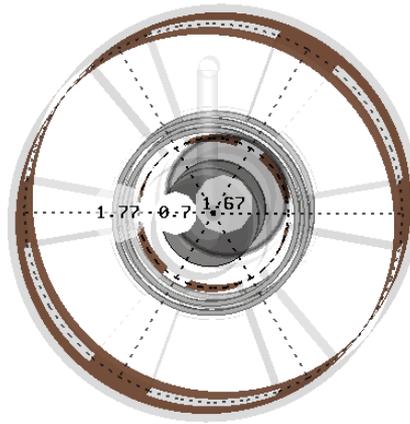
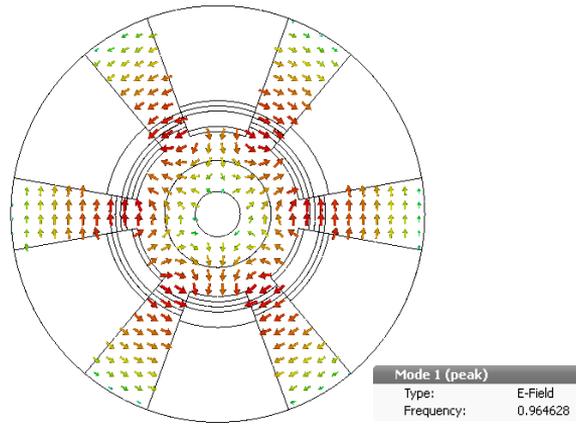
Evaluated as post
processing step



Lorentz Forces



Geometric Tolerances / Sensitivity



i Sensitivity analysis results:

Design parameter "r_a"		
Mode	Frequency	Derivative of frequency
1	0.9646199 GHz	0.1799115 GHz/in
Design parameter "r_v"		
Mode	Frequency	Derivative of frequency
1	0.9646199 GHz	-0.5956404 GHz/in

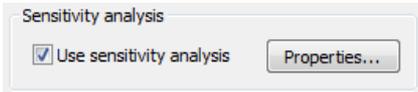
$$f_{res}(p + \Delta p) = f_{res}(p) + \frac{\partial f}{\partial p} \Delta p$$

Strapped Magnetron Example

- Face constraints applied to anode and vane radius
- Sensitivity of Eigenfrequency to (small) geometry changes in ONE 3D run

Simulating Detuning Effects

Simulating Detuning Effects

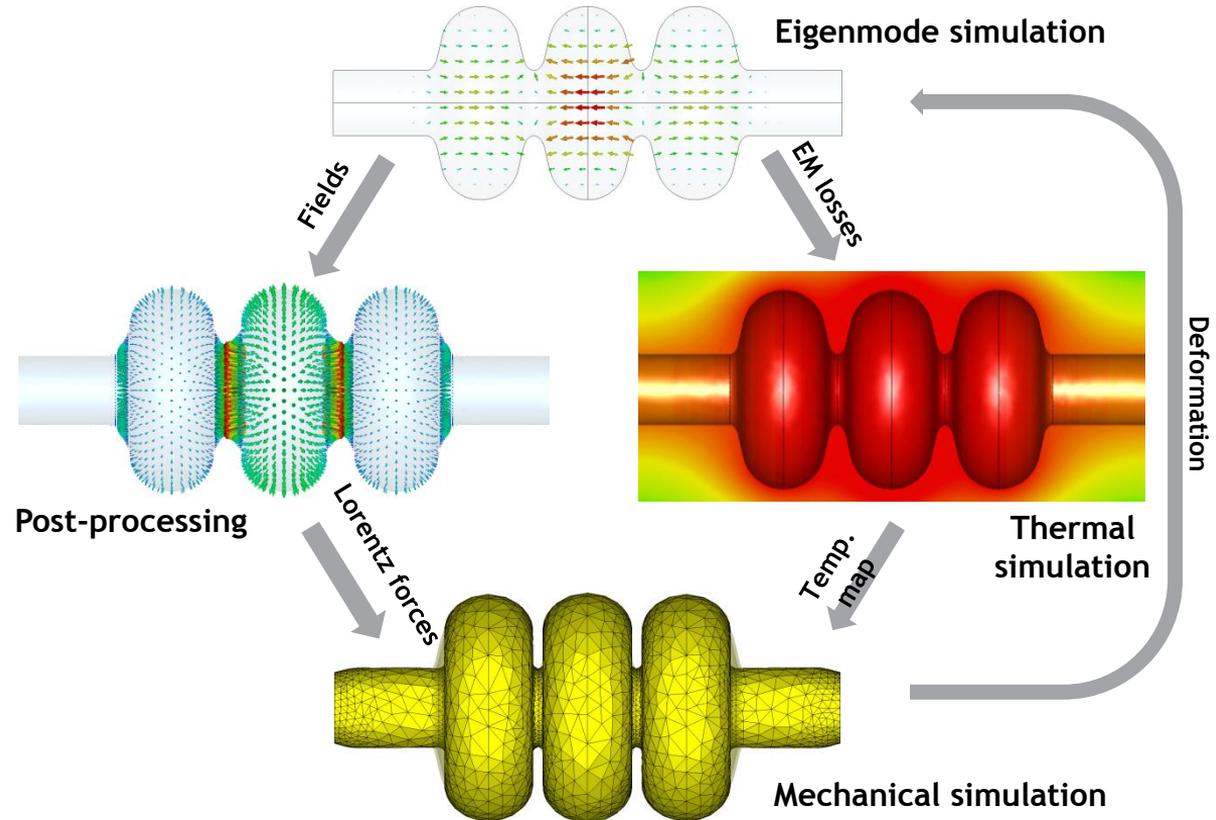
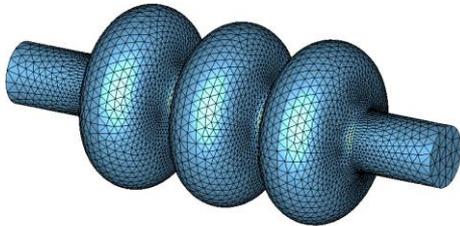


Sensitivity Analysis

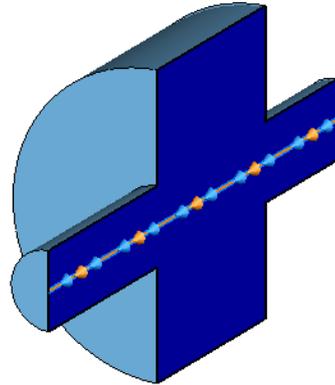
<input type="checkbox"/>	Parameter	Value	Description
<input checked="" type="checkbox"/>	field1	1	Imported displacement field

$\Rightarrow f_{initial}, \Delta f, f_{deformed}$

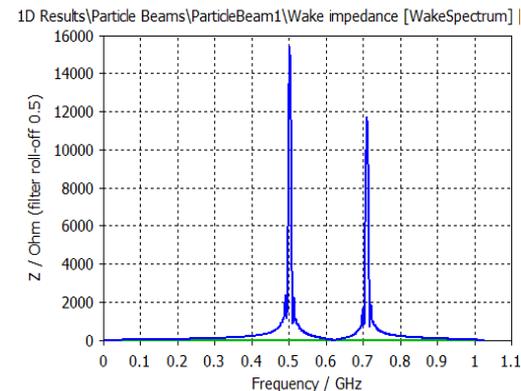
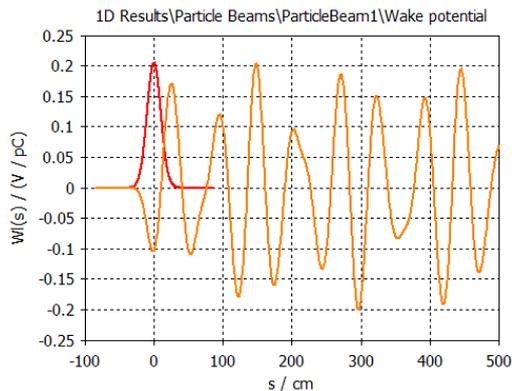
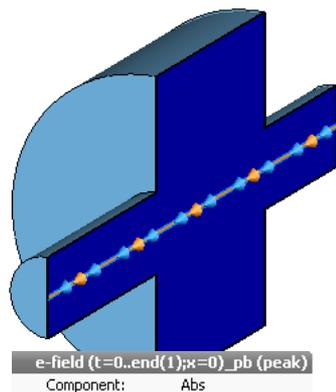
All solvers available
on tetrahedral mesh



Wakefield Analysis



Wakefields

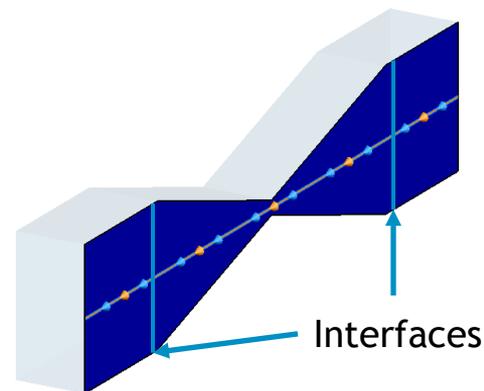
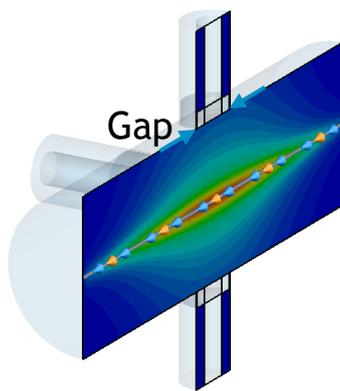
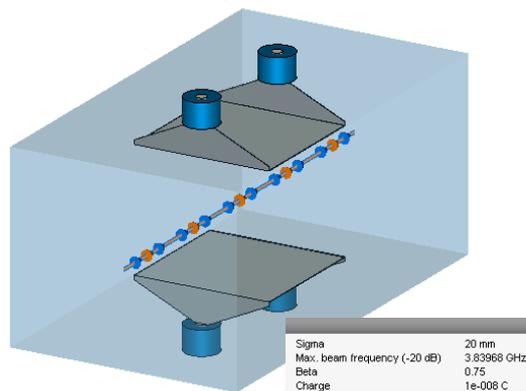


Results:

- Electromagnetic fields
- Wake Potential, Wake Impedance
- Postprocessing available for user defined sampling



Integration Methods



Direct:

- Integration along beam path
- For $\beta < 1$ the only method

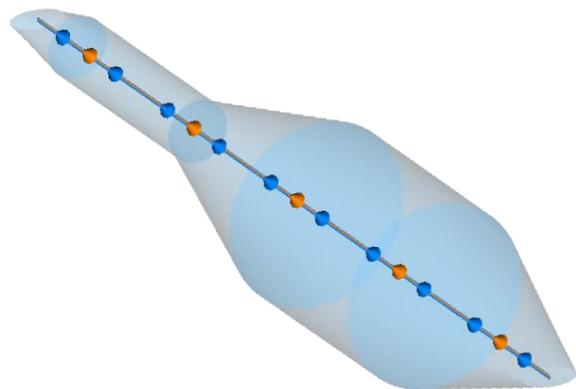
Indirect Testbeams:

- Integration along the gap
- Only for $\beta = 1$

Indirect Interfaces:

- Direct integration in discontinuity
- Tube considered on the interfaces
- Only for $\beta = 1$

Short Bunches



Total length:	1.30 m
Max. transverse dimension:	0.12 m
Beam width:	0.0001 m
⇒ Frequency:	~60GHz
⇒ Electrically Large!	

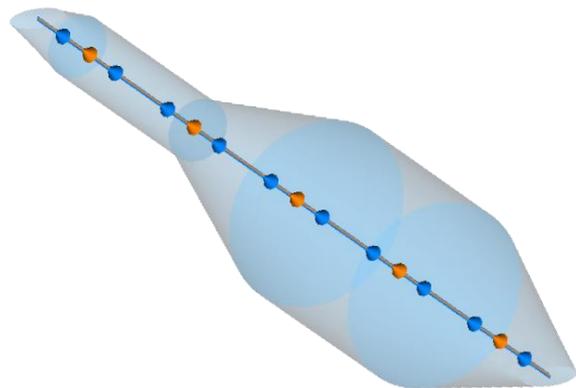
NSLS Storage Ring Impedance Budget:

→ Possible with MPI

→ Simulation Time: 5h 12min, 4 Nodes, 820 Millionen Mesh Cells

*A. Blednykh, S. Krinsky, J. Rose “Coupling Impedance of CESR- B RF Cavity for the NSLS-II Storage Ring”, BNL-79152-2007-CP

Short Bunches



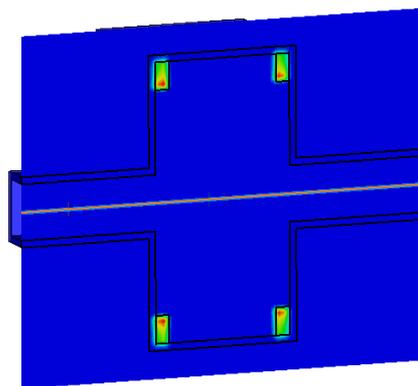
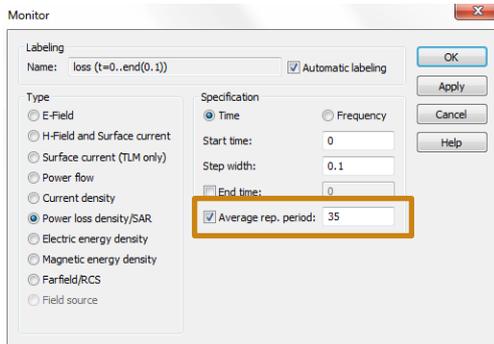
	Published	CST
Loss Factor	3.5 V/pC	3.4 V/pC
Kick Factor	25.4 V/pC/m	25 V/pC/m

NSLS Storage Ring Impedance Budget:

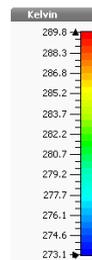
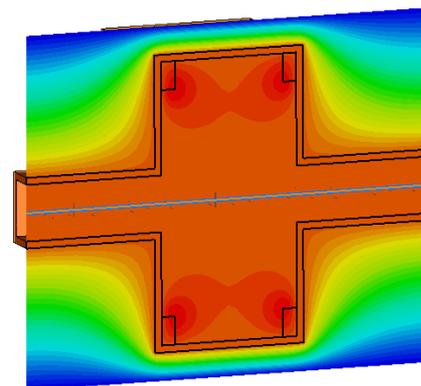
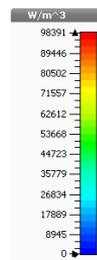
- Possible with MPI
- Simulation Time: 5h 12min, 4 Nodes, 820 Millionen Mesh Cells

*A. Blednykh, S. Krinsky, J. Rose “Coupling Impedance of CESR- B RF Cavity for the NSLS-II Storage Ring”, BNL-79152-2007-CP

Loss Consideration



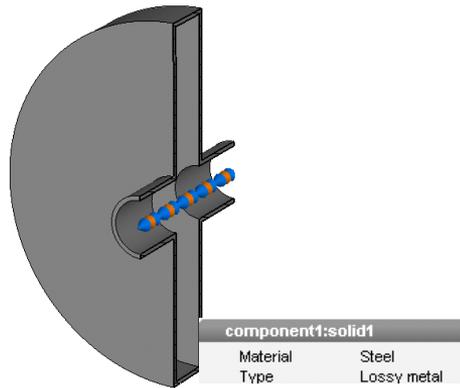
Cavity courtesy of CERN



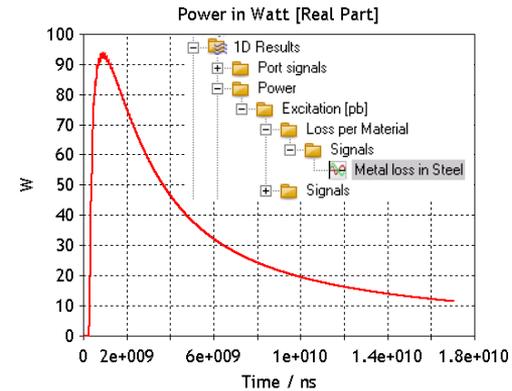
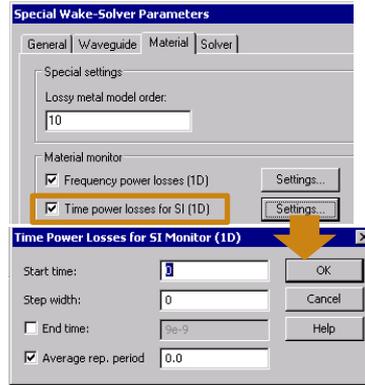
Time Integrated Volume Losses:

- 3D Information about the losses
- Coupling to thermal solver of MPS
- Averaged losses as source for thermal solver

Loss Consideration



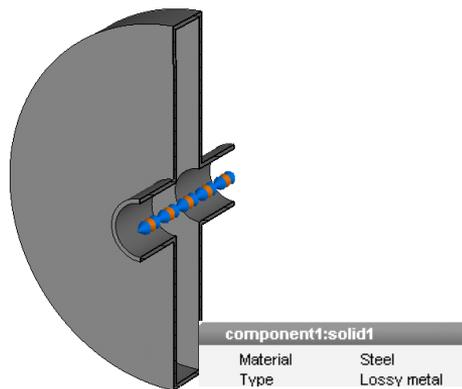
Cavity courtesy of
Diamond Light Source



Time Integrated Surface Losses:

- So far only per material
- Can be observed during simulation
- Heat source has to be applied in thermal solver

Loss Consideration

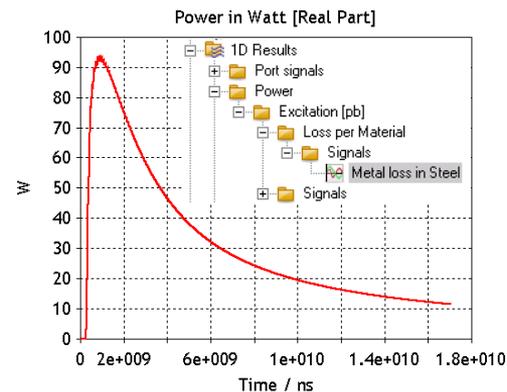


Cavity courtesy of
Diamond Light Source

Energy balance:

$$\Delta W = q^2 k$$

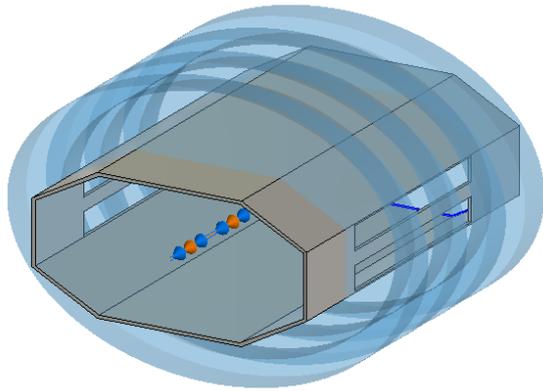
P_{loss} via k (wake loss factor)	11.29 W
P_{loss} from monitor	11.41 W



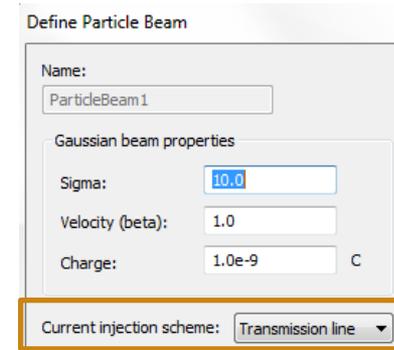
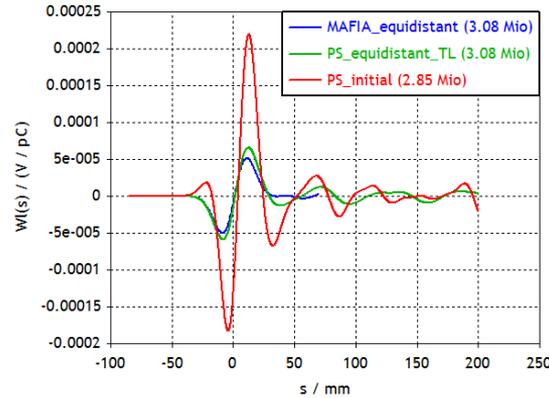
Time Integrated Surface Losses:

- So far only per material
- Can be observed during simulation
- Heat source has to be applied in thermal solver

Accuracy Consideration



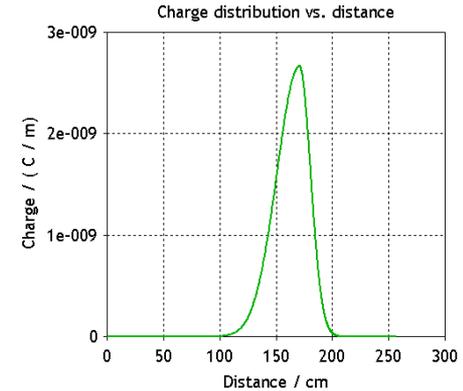
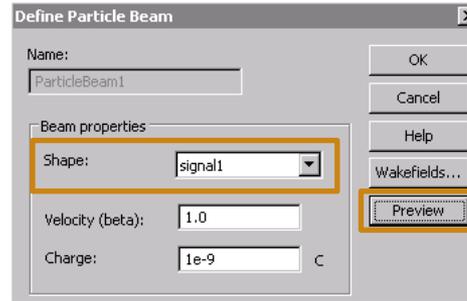
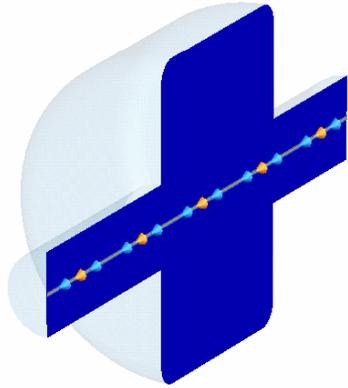
Structure courtesy of DESY



Dispersion error:

- Important, if wake potential is small
- Improves with equidistant mesh
- Improves with “Transmission line” current injection

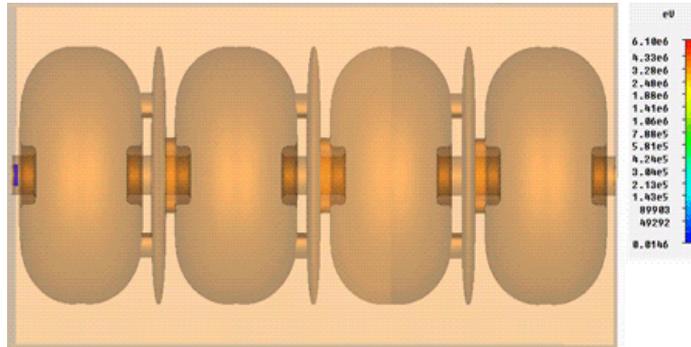
User Defined Bunch Shapes



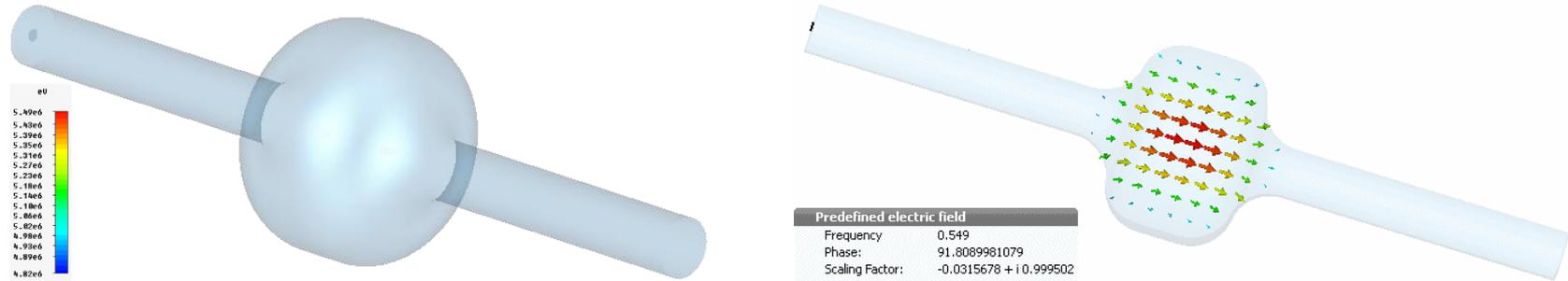
Signal definition via excitation function:

- ASCII import and formulas can be used
- Allows Non-Gaussian shapes
- Allows bunch series

Full Particle Dynamics



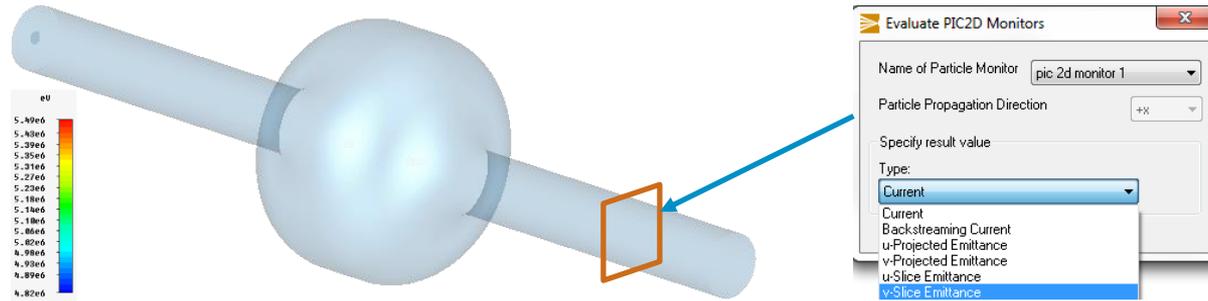
PIC Simulation of Cavities



- Preloading of Eigenmode fields
- Choice of initial phase and scaling
- Low current applications: Disregard space charge
- GPU Support: Realistic RFQ → Speed up of 4.876* and higher

* Kepler 20 Card vs. Intel E5530 2x 2.4GHz, 96GB RAM, 120k Particles, 1.36 Mio mesh cells

PIC Simulation of Cavities

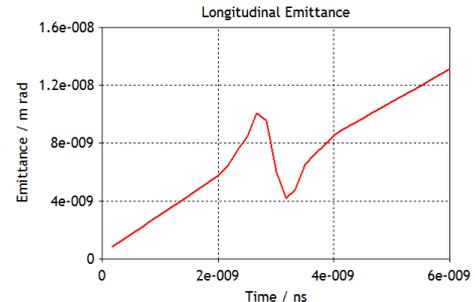
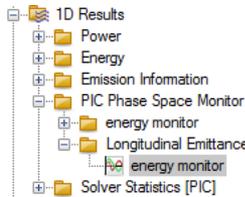


Emittance definition:

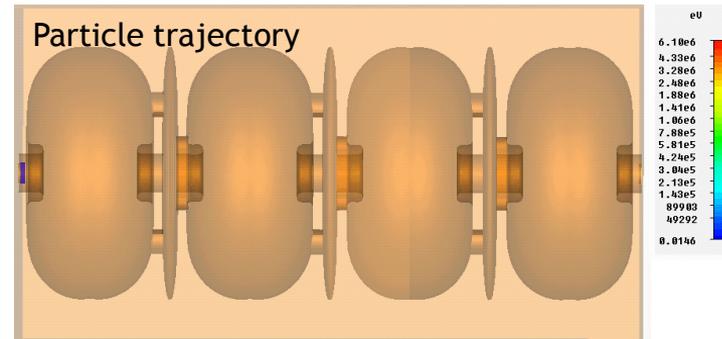
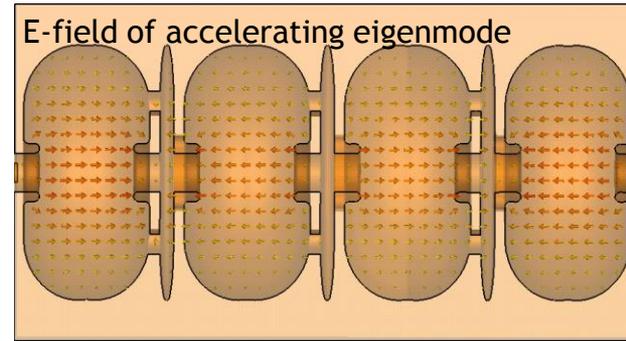
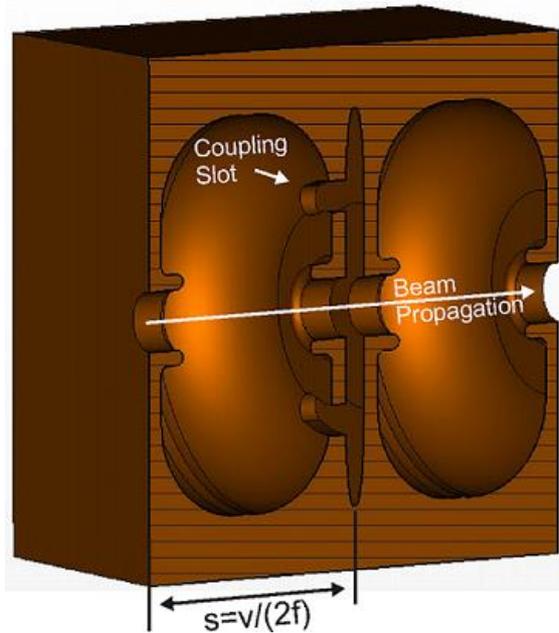
$$\epsilon_{x,rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

Postprocessing Results:

- PIC 2D monitor:
projected & slice emittance
- Phase space monitor:
longitudinal emittance

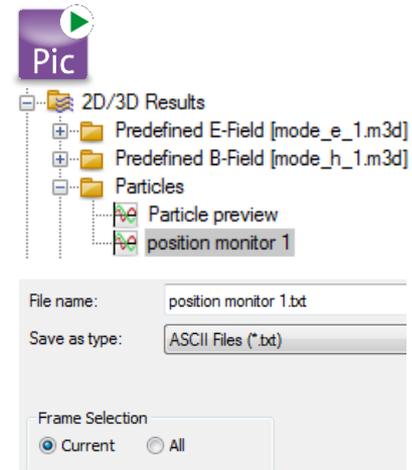
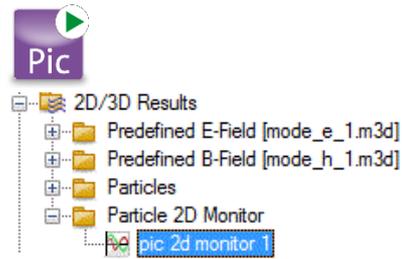
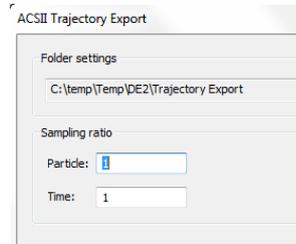
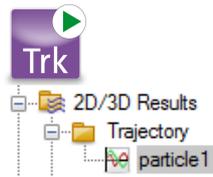


C-Band Linear Accelerator



[1] Design, simulation and measurement conducted by M. Ruf, K. Thurn and L.-P. Schmidt at Chair for High Frequency Technology, University of Erlangen-Nuremberg

Data Exchange

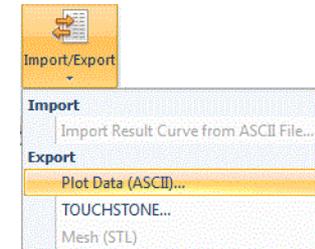


ASCII Export of:

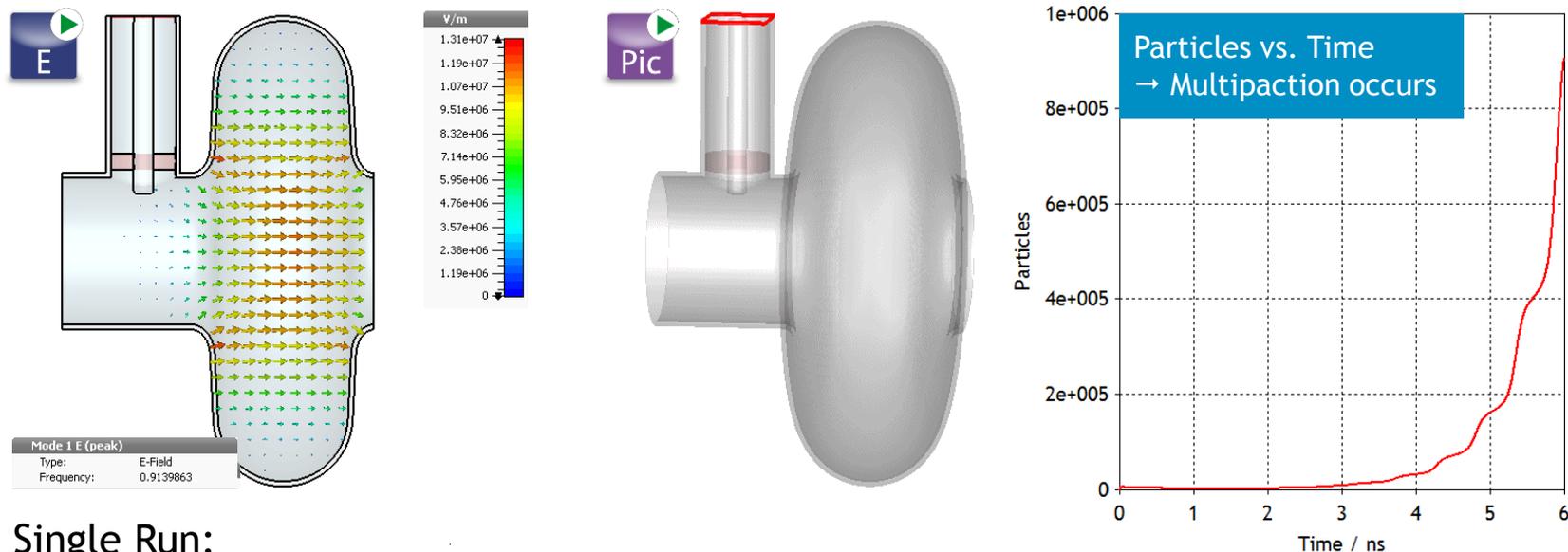
- Trajectory (TRK)
- PIC 2D / PIC Position Monitor

VBA Access of:

- Trajectory (TRK)
- PIC 2D Monitor



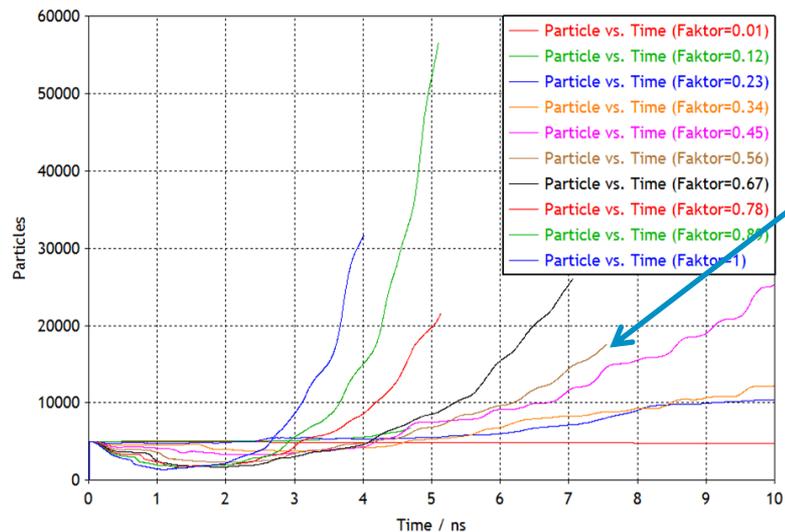
Multipaction Analysis



Single Run:

- Preloading of Eigenmode in PIC solver
- Preloading of initial particle cloud
- Observation of exponential particle increase
- Collision information available for TH analysis

Multipaction Analysis



Parametric Run:

- Reveals the power threshold
- Automatic detection of exponential increase



Automatic Solver Stopp

Multipacting

Enable solver stop

Intervals: Interval width:

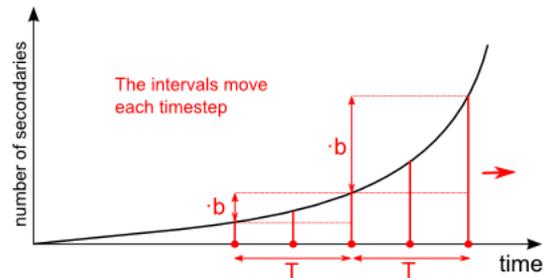
Exp. base:



PIC Special Solver Settings:

Space charge effects are neglected

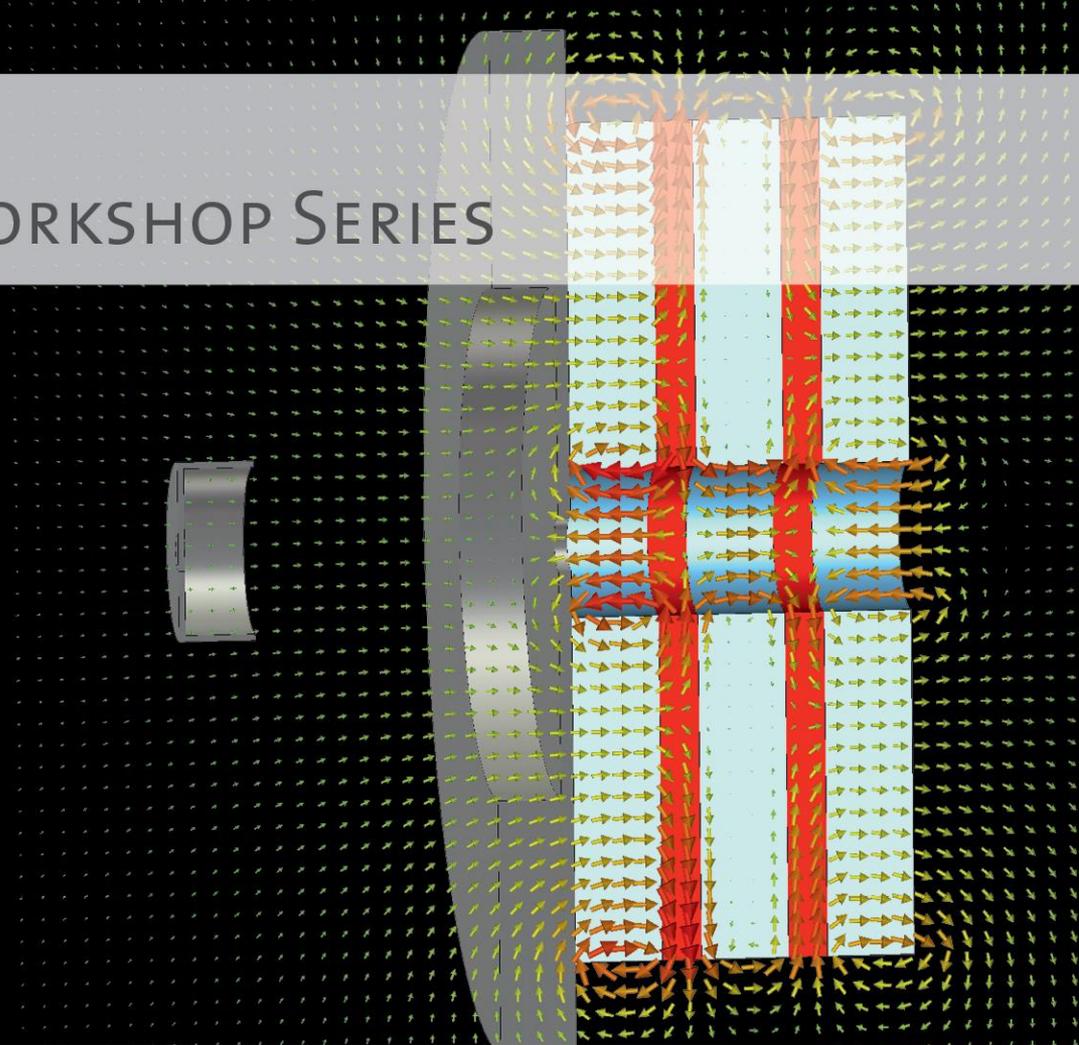
Multipacting occurred, the PIC solver finishes.



CST STUDIO SUITE®

PARTICLE DYNAMICS WORKSHOP SERIES

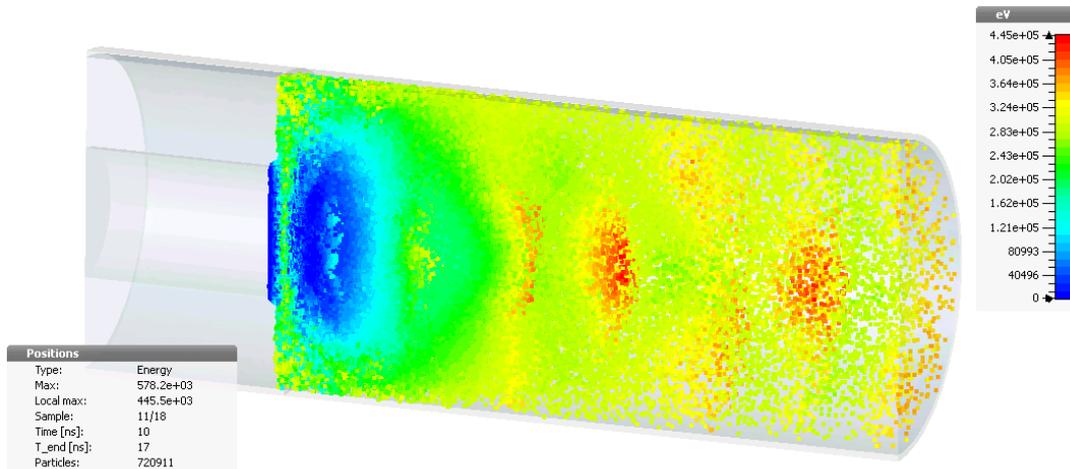
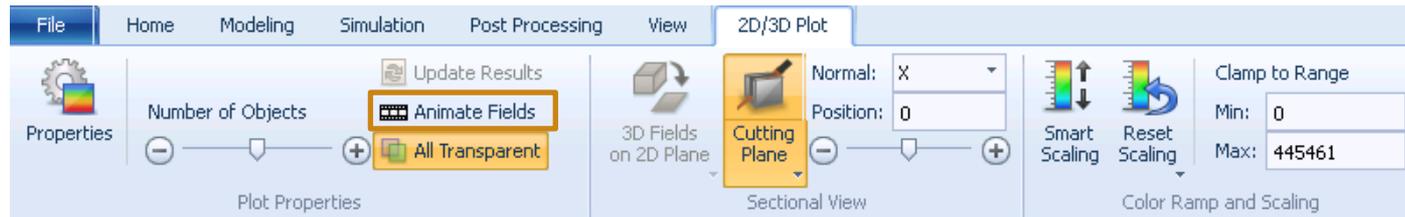
New Features 2016



Visualization



- Standard 3D Viewer now used also for PIC plots



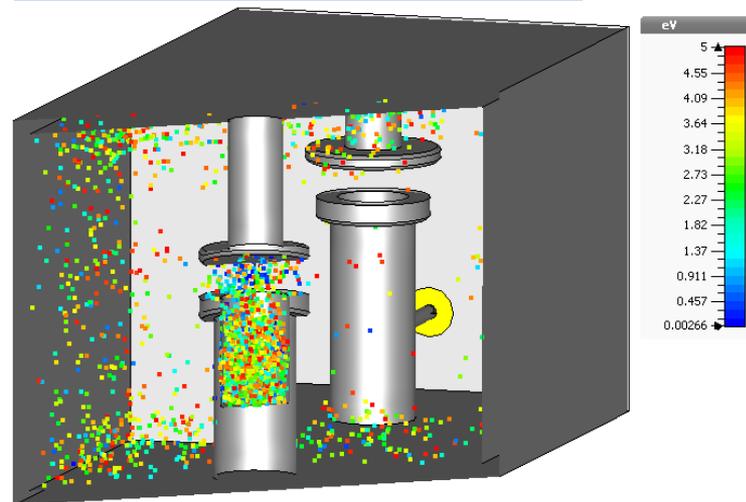
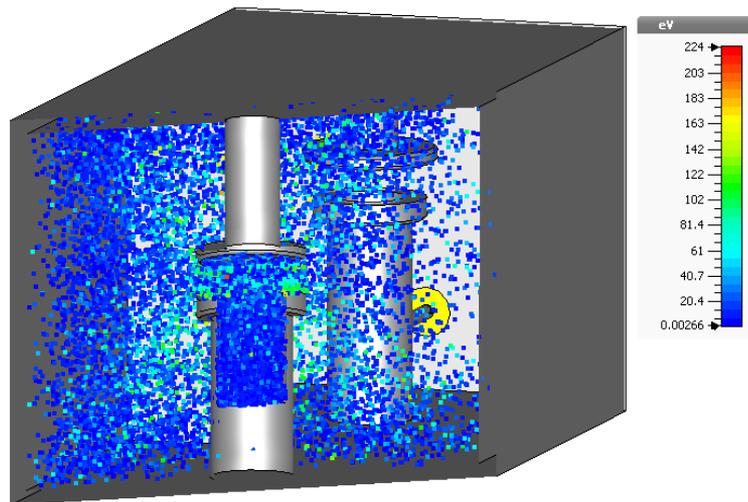
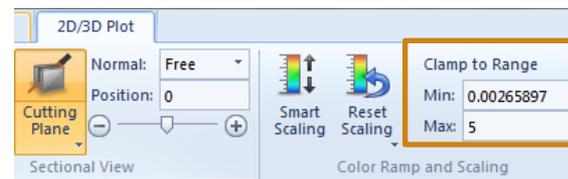
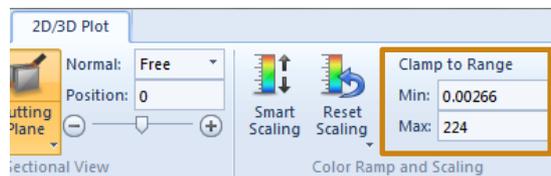
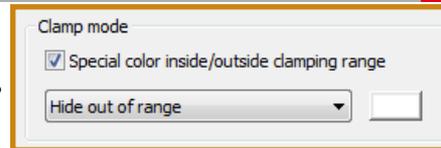
Ribbon can now be used!

Save Video Macro works as well.

Visualization



- Finding e.g. very slow particles...



Keep Mesh

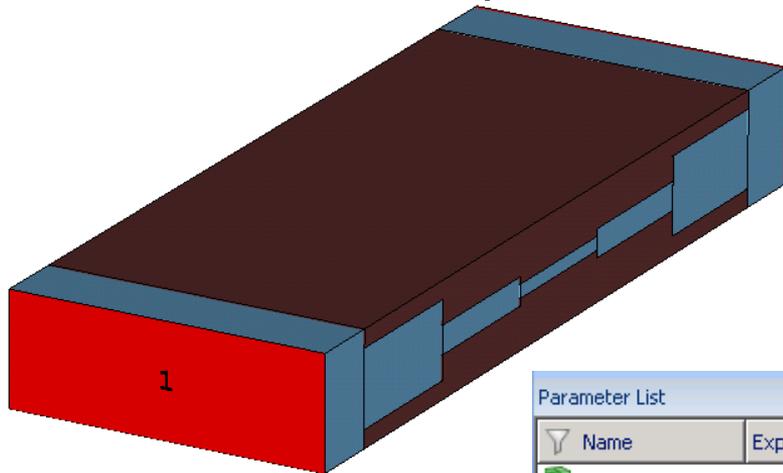


Multipaction Simulation

Parametersweep of input power

No change of geometry

Performance improvement if mesh is reused



Parameter List	
Name	Expression
peakpower	= 2.0e+004

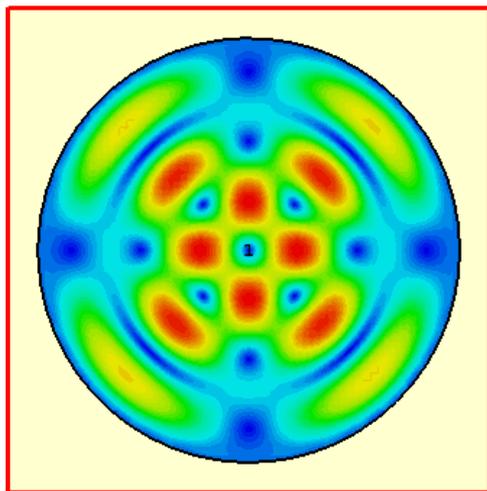
For small example:
2m 9s instead of 3m 6s

MESH PROTECTION ACTIVE:
Since only parameters which does not influence the mesh are involved, mesh will not be deleted during this Parameter Sweep run.

<input type="checkbox"/> Electric Field	Factor: 1.0	Settings...
<input type="checkbox"/> Magnetic Field	Factor: 1.0	Settings...
<input type="checkbox"/> Analytic Field	Factor: 1.0	Settings...
<input checked="" type="checkbox"/> External Field	Factor: <code>sqr(peakpower)</code>	Settings...

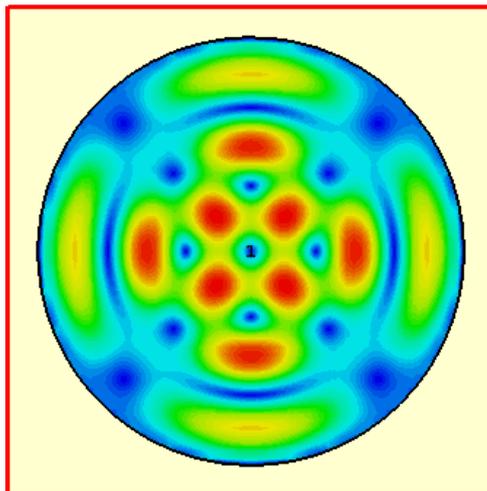
Apply
Close
Help

Target Frequency for WG Port



Port1_e51 (peak)	
Component:	Abs
Fcutoff:	33.87
Mode type:	TE

Mode #51

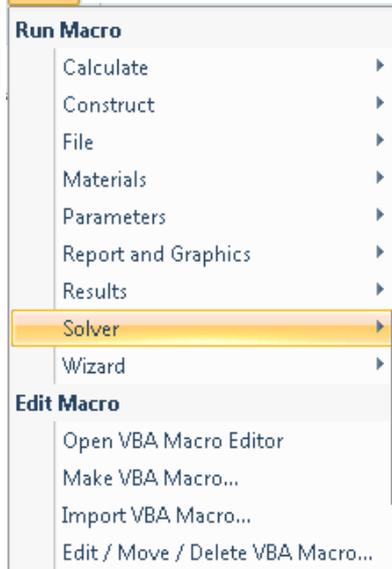
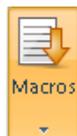


Port1_e3[42] (peak)	
Component:	Abs
Fcutoff:	33.63
Mode type:	TE

Mode #3

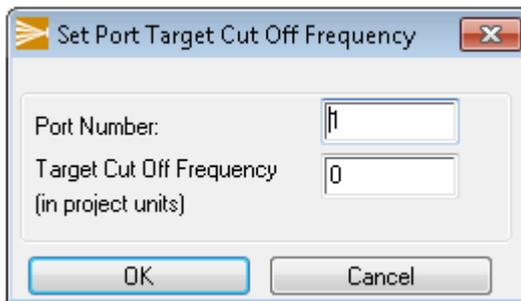
- Reduction of evaluated port modes
- Absorption of unconsidered port modes
- Reduction in total simulation time by factor 8.7

Target Frequency for WG Port



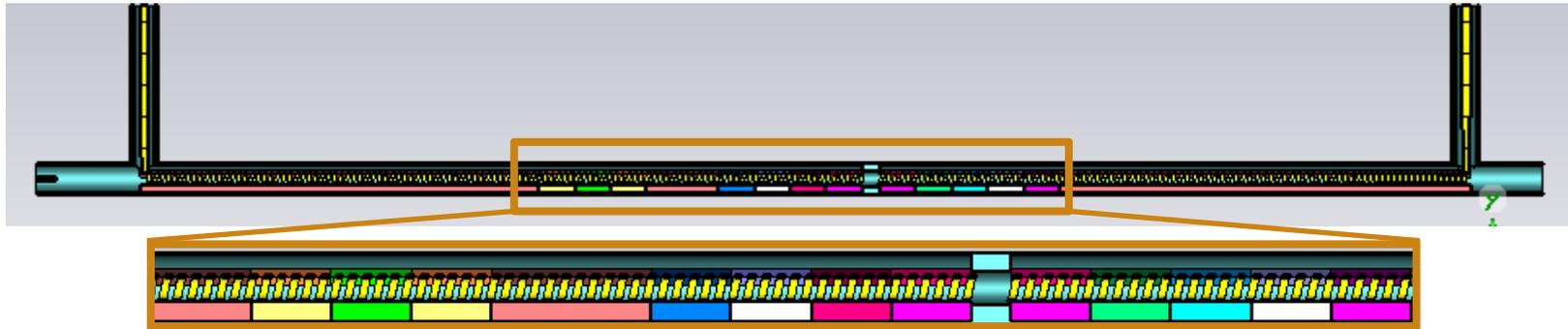
Easier for the user:

- no VBA definition
- automatic setting of generalized port mode solver

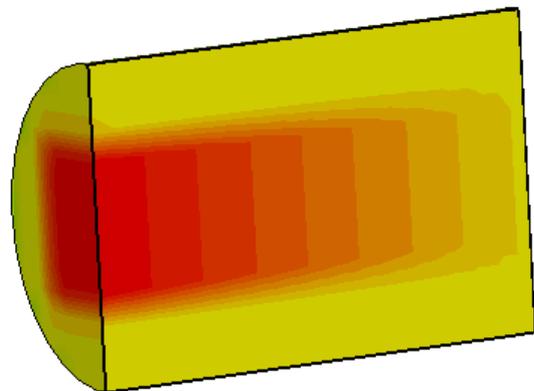
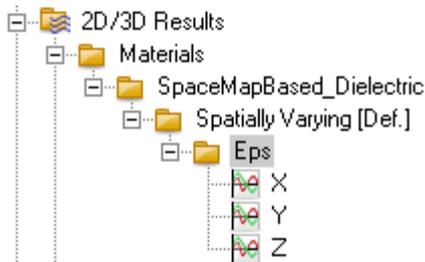


Spatially Varying Material

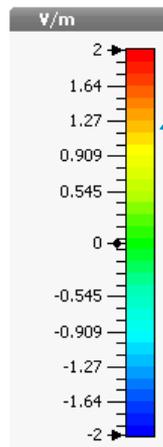
- Axial varying losses for e.g. TWT can now be applied to one single material by space map based material
- Splitting in different object/materials is obsolete now



Spatially Varying Material



Eps
Cutplane Normal: 1, 0, 0
Cutplane Position: 0



Loaded from Textfile
via VBA

```
MaterialMap_eps - Notepad
File Edit Format View Help
# CST material field file
# Format: FixedGrid
# Version: 20150107

# LengthUnit: mm

# SamplePoint x:
0.0 15.0

# SamplePoint y:
0.0 15.0

# SamplePoint z:
20.0 100.0

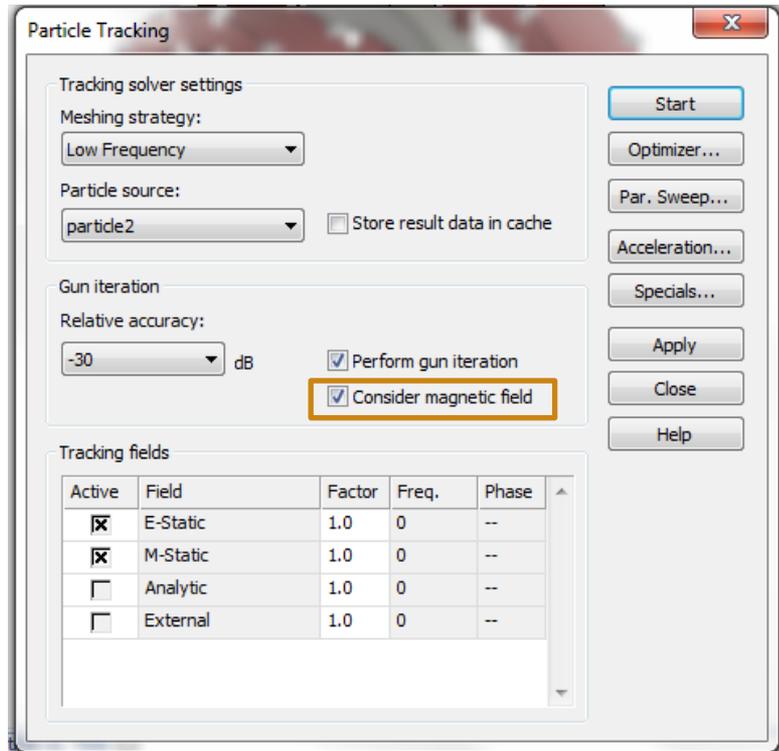
# Symmetry x: true
# Symmetry y: true

# Data section:

1 1
1 1

2 2
2 2
```

Eigen H-Field in Gun Iteration

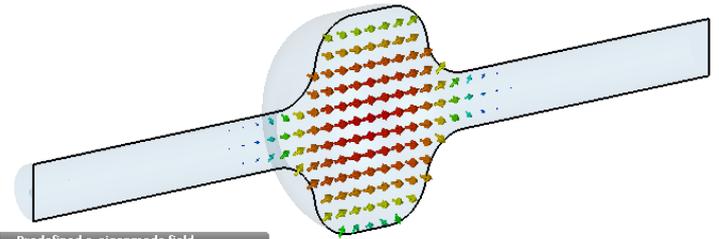
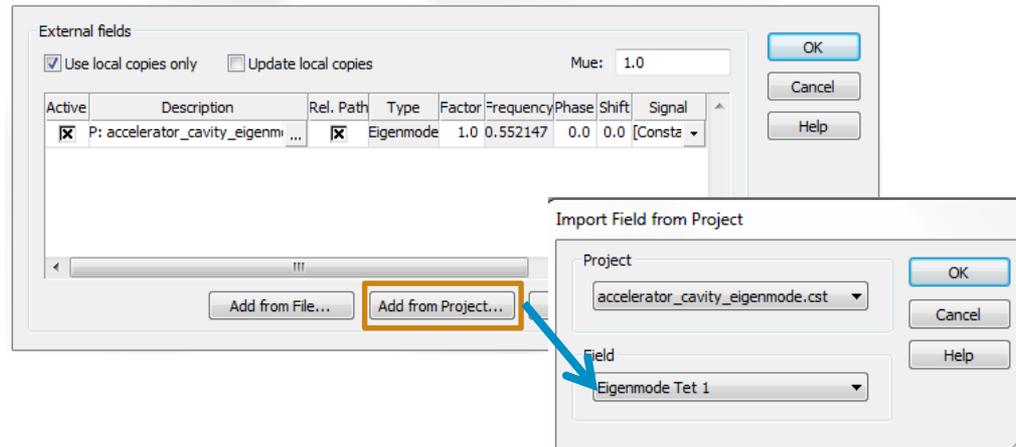


Important for High Voltage guns (~500kV), where the Eigen H-Field has a focusing effect in contrast to the Eigen E-Field.

Otherwise beam expansion is overestimated.

External TET Field Import

Import External Fields

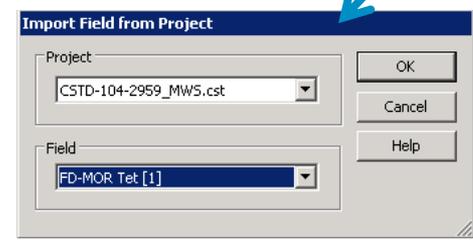
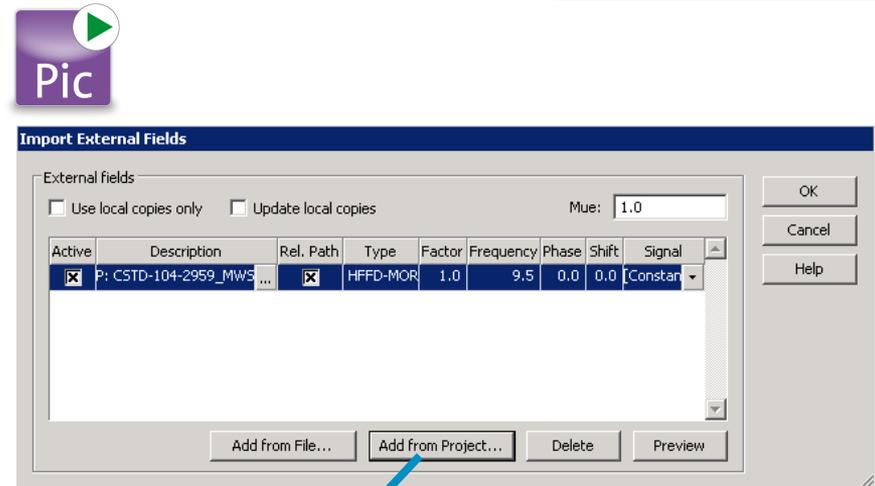
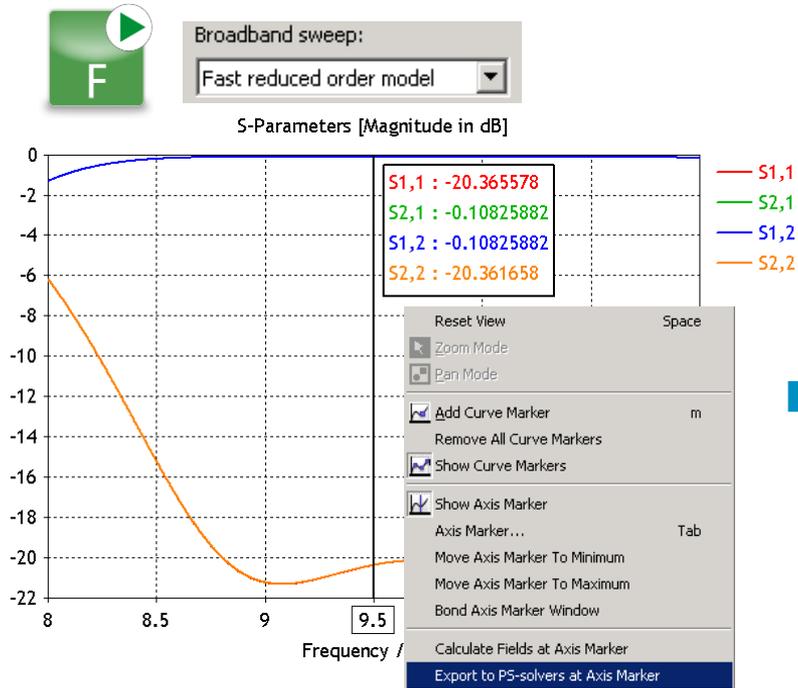


Predefined e-eigenmode field
Cutplane name: Cross Section A
2D Maximum: 4.24e+06
Cutplane normal: 1, 0, 0
Cutplane position: 0
Frequency: 0.5521
Phase: 67.5
Scaling Factor: -0.0315678 + i -0.999502

TET Fields can be loaded directly from

- E-Static
- M-Static
- Eigenmode

External TET Field Import



- Solutions for 2 excitations were exported.
- Frequency Domain Solver successful

Spectrogram



Frequency vs. Time Behaviour for Non-linear Systems

Fourier Transforms

Specify Action

Fourier Transform

Resample with Nyquist rate before evaluation

To single-sided spectrum

Fmin: 0.3

Fmax: 8

of samples: [Auto, from length of input signal]

Normalization: 1

Result type: Magnitude (linear)

Time Window Settings

1D Results

Probes\E-Field\Probe Signals\E-field (2.11 0.0 3.2)(Y) [pic]

Complex Re Im Mag MagdB Ph

field (2.11 0.0 3.2)(Y) [pic_FT\center = 2]

Complex Re Im Mag MagdB Ph

OK Cancel Help

Time Window Settings

Window function: Rect

Window width in project units: 2

Sweep window center:

From t0 = 2 to tN = 18

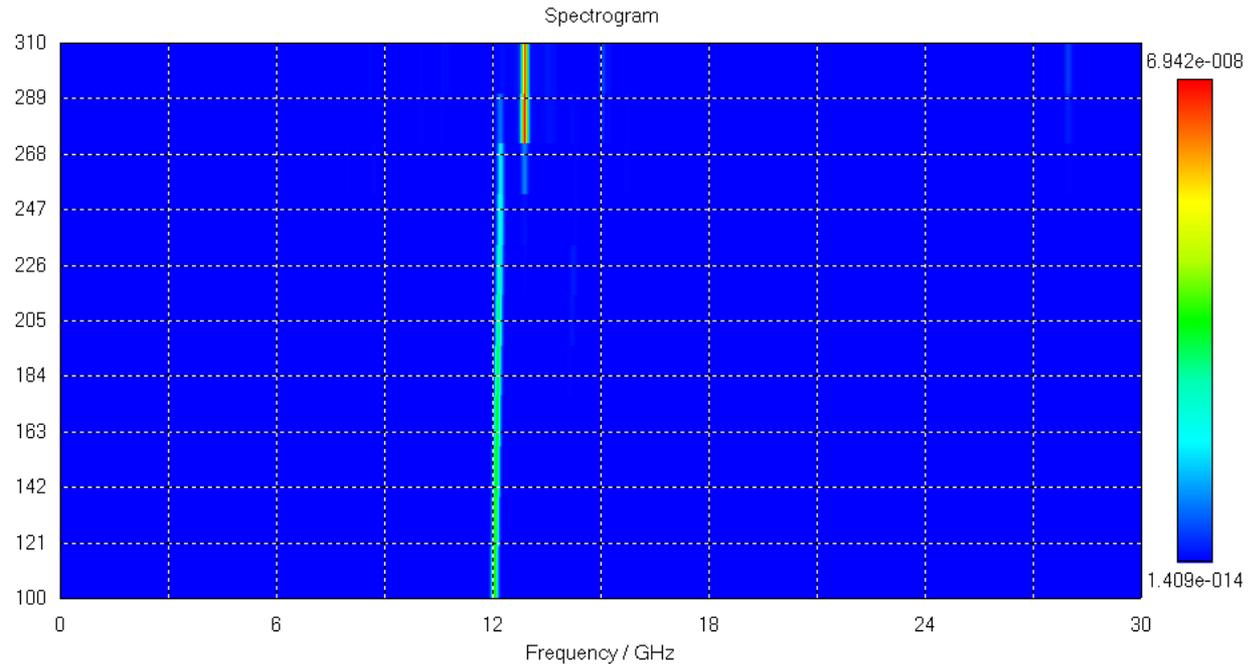
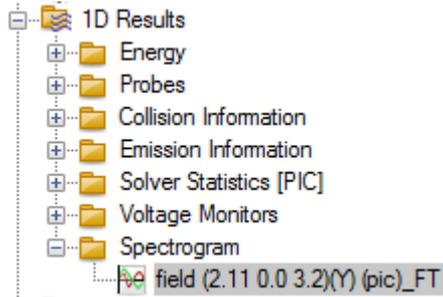
using 9 time step(s).

OK

Spectrogram



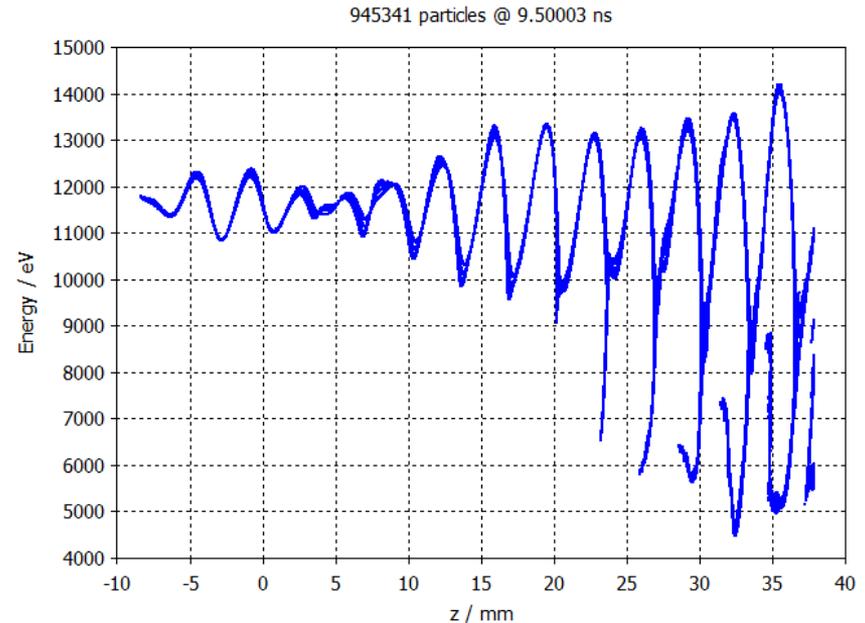
Frequency vs. Time Behaviour for Non-linear Systems



Phase Space Monitor



- Performance Improvement
 - Faster resize
 - Faster plot option change



Phase Space Monitor



- More Flexibility
 - Abscissa and ordinate can be chosen from:

- Position
- Normed Momentum
- Gamma
- Beta
- Velocity
- Energy

The dialog box "Define PIC Phase Space Monitor" contains the following settings:

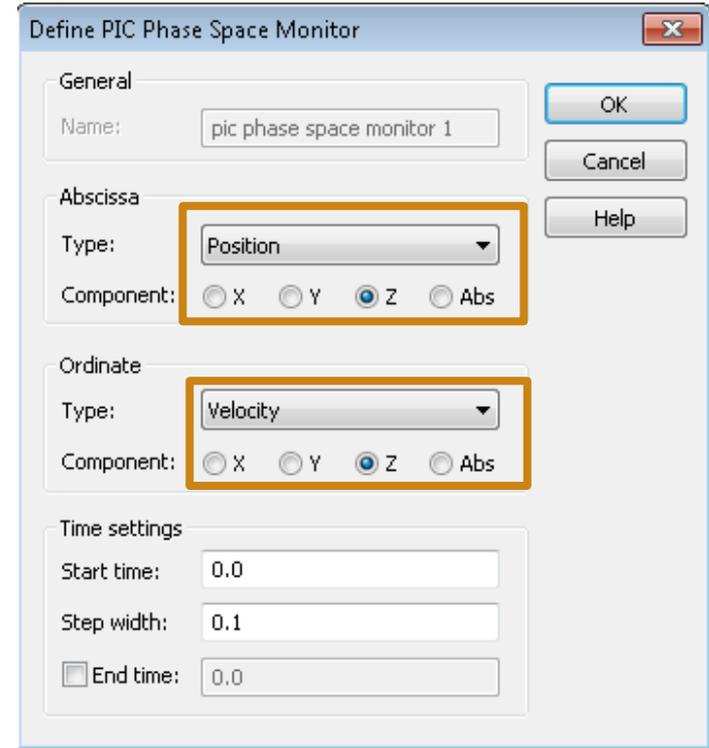
- General:** Name: pic phase space monitor 1
- Abscissa:** Type: Position; Component: X Y Z Abs
- Ordinate:** Type: Energy; Component: X Y Z Abs
- Time settings:** Start time: 0.0; Step width: 0.1; End time: 0.0

Buttons: OK, Cancel, Help

Phase Space Monitor

- More Flexibility
 - Abscissa and ordinate can be chosen from:

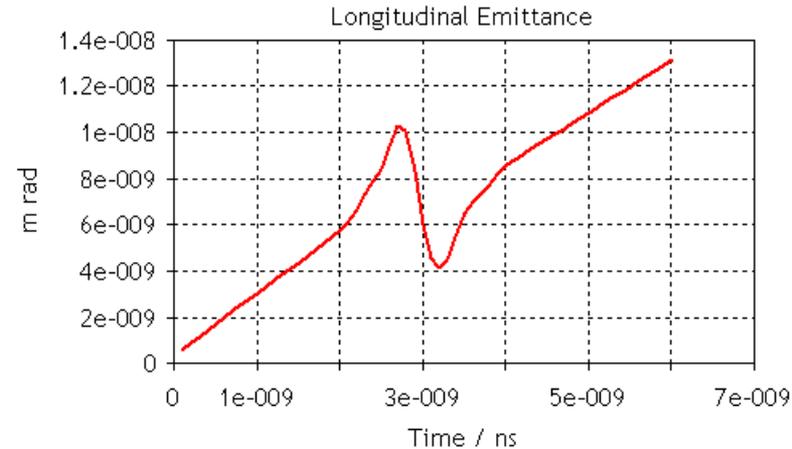
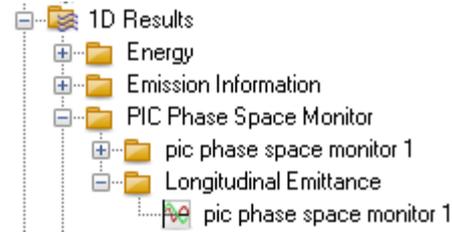
Position
 Normed Momentum
 Gamma
 Beta
 Velocity
 Energy
 - Special Case:
 Equal Position and Velocity
 componente → Evaluation of
 longitudinal Emittance



Phase Space Monitor



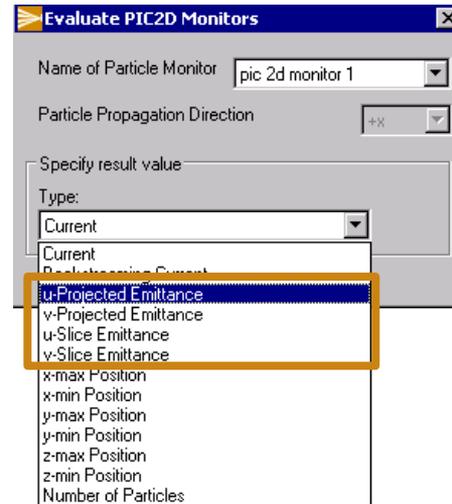
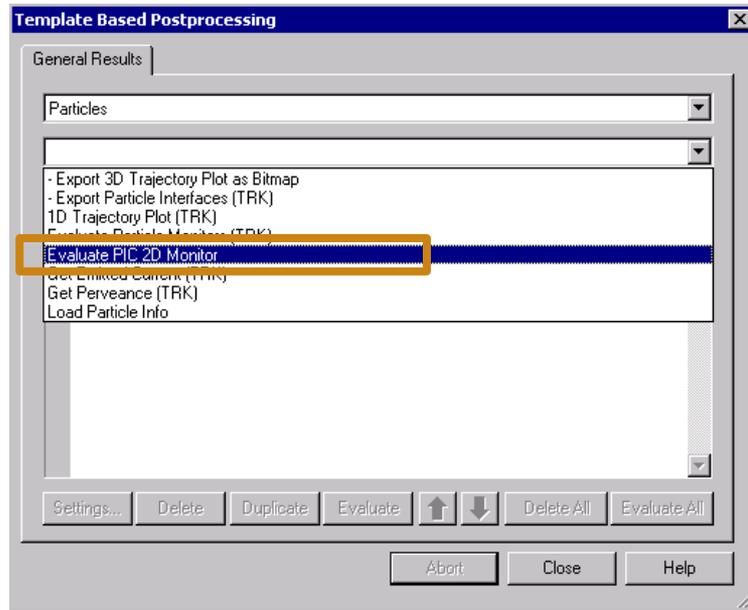
- More Flexibility
 - Abscissa and ordinate can be chosen from:
 - Position
 - Normed Momentum
 - Gamma
 - Beta
 - Velocity
 - Energy
- Special Case:
Equal Position and Velocity
componente → Evaluation of
longitudinal Emittance



Emittance from PIC 2D Monitor



- Projected as well as slice emittance



Emittance from PIC 2D Monitor



Please note:

Emittance implemented in TRK code & Template

$$\epsilon_{x,rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

Emittance often used by accelerator people

The normalised emittance is given by $\epsilon_{n,rms} = \beta\gamma\epsilon_{rms}$, where the two factors are the relativistic Lorentz factors. The normalisation is necessary to make beams with different energies comparable [6].

TET TRK Solver

